Algebra	
A1	Fix a positive integer d. Yael and Ziad play a game as follows, involving a monic polynomial of degree $2d$. With Yael going first, they take turns to choose a strictly positive real number as the value of one of the coecients of the polynomial. Once a coefficient is assigned a value, it cannot be chosen again later in the game. So the game lasts for $2d$ rounds, until Ziad assigns the final coefficient. Yael wins if $P(x) = 0$ for some real number x. Otherwise, Ziad wins. Decide who has the winning strategy.
A2	Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) + f(y) + f(z) = f(xyz)$ for any real numbers x, y, z with $x + y + z = 1$.
A3	For a positive integer n denote $F_n(x_1, x_2, \ldots, x_n) = 1 + x_1 + x_1x_2 + \cdots + x_1x_2 \ldots x_n$. For any real numbers $x_1 \geq x_2 \geq \ldots \geq x_k \geq 0$ prove that $$\prod_{i=1}^{k} F_i(x_{k-i+1}, x_{k-i+2}, \ldots, x_k) \geq \prod_{i=1}^{k} F_i(x_i, x_i, \ldots, x_i)$$
A4	Alice and Bob play the following game. Bob chooses 2024 polynomials with real coefficients, $P, P_1, \ldots, P_{2023}$, such that $1 < \deg(P) < \deg(P_i), i = 1, 2, \ldots, 2023$, and shows them all to Alice. At each step $i \geq 1$, Alice chooses a real number $a_i > 0$ such that $a_{i+1} \geq a_i + 2024$ and shows Bob a_i. Then either he chooses a real number b_i satisfying $P_j(b_i) = P(a_i)$ for some j or, if no such a b_i exists, he chooses $b_i = a_i$. Alice wins if, at some stage, she can exhibit distinct integers k and m and a real number c satisfying $\lvert b_{k+i} + c - b_{m+i} \rvert \leq 10^{-2024}$, for all $i = 1, 2, \ldots, 2024$. Otherwise, Bob wins. Decide who has a winning strategy. (BULGARIA, *Dragomir Grozev*)

Geometry	
G1	Let ABC be an acute triangle with $\angle ABC > 45^\circ$ and $\angle ACB > 45^\circ$. Let M be the midpoint of the side BC. The circumcircle of triangle ABM crosses the side AC again at X and the circumcircle of triangle AMC crosses the side AB again at Y. The point P lies on the perpendicular bisector of the segment BC, so that the points A and P lie on the same side of XY, and $\angle YPX = 90^\circ + \angle BAC$. Prove that the circumcircles of triangles BYP and CXP are tangent.
G2	Let ABC be an acute triangle with orthocentre H and circumcircle Γ. Let D be the point diametrically opposite A on Γ. The line through H, parallel to BC, intersects AB and AC at X and Y, respectively. Let AD intersect the circumcircle of triangle DXY again at S. Let the tangent to Γ at A intersect XY at T. Prove that lines DT and HS intersect on Γ.

Number Theory	
N1	Determine all infinite sequences $a_1, a_2, \ldots$ of positive integers, such that $$2024\left(a_{n+1}^3 + 1\right) = (2024a_na_{n+1} + 1)(a_{n+2} + 2023)$$ for any positive integer n.
N2	Let a_1 be a positive integer and $a_n = \sum_{i=1}^{n-1} \gcd(n, a_i)$ for any $n \geq 2$. Prove that $a_{n+1} \leq a_n$ for infinitely many n.

Combinatorics	
C1	Fix an integer $n \geq 2$. Consider $2n$ real numbers $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$. Let S be the set of all pairs (x, y) of real numbers for which $M_i = a_i x + b_i y$, $i = 1, 2, \ldots, n$ are pairwise distinct. For every such pair sort the corresponding values $M_1, M_2, \ldots, M_n$ increasingly and let $M(i)$ be the i-th term in the list thus sorted. This defines an index permutation of $1, 2, \ldots, n$. Let N be the number of all such permutations, as the pairs run through all of S. In terms of n, determine the largest value N may achieve over all possible choices of $a_1, \ldots, a_n, b_1, \ldots, b_n$.
C2	Fix an integer $n \geq 3$ and let $A_1 A_2 \ldots A_n$ be a convex polygon in the plane. Let $\mathcal{M}$ be the set of all midpoints $M_{i,j}$ of segments $A_i A_j$ where $i \neq j$. Assume that all of these midpoints are distinct, i.e. $\mathcal{M}$ consists of $\dfrac{n(n-1)}{2}$ elements. Dissect the polygon $M_{1,2} M_{2,3} \ldots M_{n,1}$ into triangles so that the following hold: (1) The intersection of every two triangles (interior and boundary) is either empty or a common vertex or a common side. (2) The vertices of all triangles lie in M (not all points in M are necessarily used). (3) Each side of every triangle is of the form $M_{i,j} M_{i,k}$ for some pairwise distinct indices i, j, k. Prove that the total number of triangles in such a dissection is $3n - 8$.
C3	Fix an odd integer $n \geq 3$. At a maths camp, there are n^2 children, each of whom selects either algebra or geometry as their favourite topic. At lunch, they sit at n tables, with n children on each table, and start talking about mathematics. A child is said to be popular if their favourite topic has a majority at their table. For dinner, the students again sit at n tables, with n children on each table, such that no two children share a table at both lunch and dinner. Determine the minimal number of young mathematicians who are popular at both mealtimes. (The minimum is across all sets of topic preferences and seating arrangements.)
C4	Let n be a positive integer. For a set S of n real numbers, let $f(S)$ denote the number of increasing arithmetic progressions of length at least two all of whose terms are in S. Prove that, if S is a set of n real numbers, then $$f(S) \leq \frac{n^2}{4} + f(\{1, 2, \ldots, n\})$$