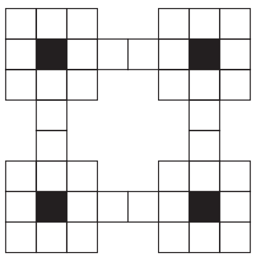
JUNIOR (grades 9 and 10)

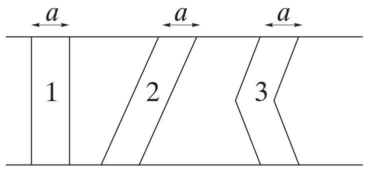
3-POINT QUESTIONS

J1. 15% of a round cake is cut as shown in the figure. How many degrees is the angle denoted by the question mark?
A 30° **B** 45° **C** 54° **D** 15° **E** 20°

J2. The composite board shown in the picture consists of 44 fields 1×1. How many possibilities are there to cover all 40 white fields with 20 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 8 **B** 16 **C** 32 **D** 64 **E** 100



J3. In the picture, three strips of the same horizontal width a are marked 1, 2, 3. These strips connect the two parallel lines. Which strip has the biggest area?
A All three strips have the same area
B Strip 1 **C** Strip 2 **D** Strip 3
E Impossible to answer without knowing a



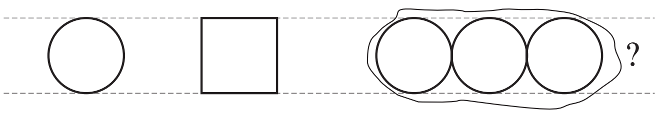
J4. Which of the following numbers is odd for every integer n?
A $2003n$ **B** n^2+2003 **C** n^3 **D** $n+2004$ **E** $2n^2+2003$

J5. In a triangle ABC the angle C is three times bigger than the angle A, the angle B is two times bigger than the angle A. Then the triangle ABC
A is equilateral **B** is isosceles **C** has an obtuse angle **D** has a right angle
E has only acute angles

J6. Three singers take part in a musical round of 4 equal lines, each finishing after singing the round 3 times. The second singer begins the first line when the first singer begins the second line, the third singer begins the first line when the first singer begins the third line. The fraction of the total singing time that all three are singing at the same time is
A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{4}{7}$ **D** $\frac{5}{7}$ **E** $\frac{7}{11}$

J7. The number $a = 111\ldots111$ consists of 2003 digits, each equal to 1. What is the sum of the digits of the product $2003 \cdot a$?
A 10 000 **B** 10 015 **C** 10 020 **D** 10 030 **E** 2003^2

J8. The area of the wooden square equals a. The area of each wooden circle equals b. Three circles are lined up as shown in the picture. If we tie together the three circles with a thread as short as possible, without moving them, what is the area inside the thread?



A $3b$ **B** $2a+b$ **C** $a+2b$ **D** $3a$ **E** $a+b$

J9. How many of the functions $f(x)=0$, $f(x)=\frac{1}{2}$, $f(x)=1$, $f(x)=x$, $f(x)=-x$ satisfy the equation $f(x^2+y^2)=f^2(x)+f^2(y)$?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

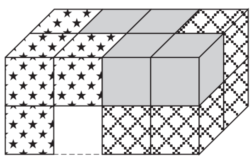
J10. In this addition each of the letters X, Y and Z represents a different non-zero digit. The letter X will then have to stand for
A 1 **B** 2 **C** 7 **D** 8 **E** 9

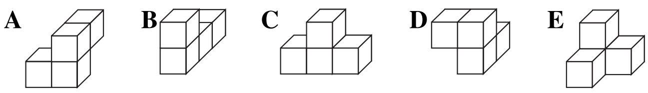
$$\begin{array}{r} XX \\ +\,YY \\ ZZ \\ \hline ZYX \end{array}$$

4-POINT QUESTIONS

J11. Ann has a box containing 9 pencils. At least one of them is blue. Among every 4 of the pencils at least two have the same colour, and among every 5 of the pencils at most three have the same colour. What is the number of blue pencils?
A 2 **B** 3 **C** 4 **D** 1 **E** Impossible to determine

J12. A rectangular parallelepiped was composed of 4 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?

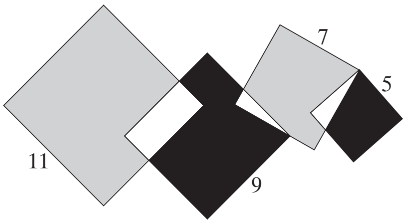




J13. When a barrel is 30% empty it contains 30 litres more when it is 30% full. How many litres does the barrel hold when full?
A 60 **B** 75 **C** 90 **D** 100 **E** 120

J14. Each of two pupils changed two of the digits of 3-digit number 888 and got a new 3-digit number which is still divisible by 8. What is the biggest possible difference of their numbers?
A 800 **B** 840 **C** 856 **D** 864 **E** 904

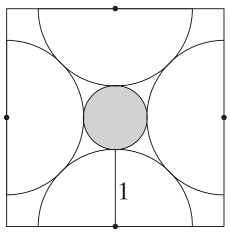
J15. In the picture there are four overlapping squares with sides 11, 9, 7 and 5 long. How much greater is the sum of the two grey areas than the sum of the two black areas?
A 25 **B** 36 **C** 49 **D** 64 **E** 0



J16. The value of the product

$$\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{2003}\right)$$

is equal to
A 2004 **B** 2003 **C** 2002 **D** 1002 **E** 1001



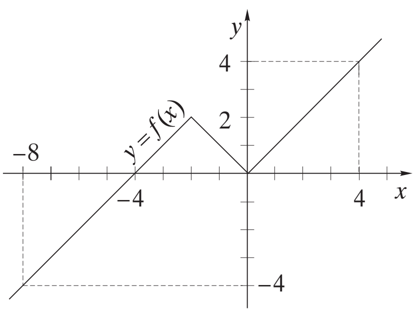
J17. The diagram shows four semicircles with radius 1. The centres of the semicircles are at the mid-points of the sides of a square. What is the radius of the circle which touches all four semicircles?

A $\sqrt{2}-1$ **B** $\frac{\pi}{2}-1$ **C** $\sqrt{3}-1$ **D** $\sqrt{5}-2$ **E** $\sqrt{7}-2$

J18. Consider all the different four-digit numbers that you can form by using the four digits of the number 2003. Summing up all them (including 2003 itself) you get:
A 5 005 **B** 5 555 **C** 16 665 **D** 1 110 **E** 15 555

J19. The first two terms of a sequence are 1 and 2. Each next term is obtained by dividing the term before the previous one by the previous term. What is the tenth term of this sequence?
A 2^{-10} **B** 256 **C** 2^{-13} **D** 1024 **E** 2^{34}

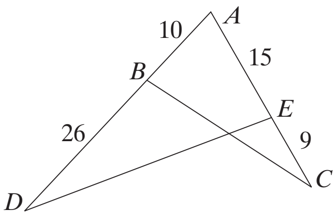
J20. The graph of the function $f(x)$, defined for all real numbers, is formed by two half-lines and one segment, as illustrated in the picture.



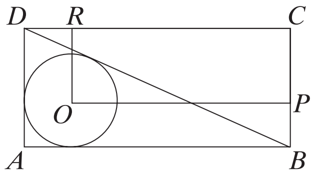
Clearly, -8 is a solution of the equation $f(f(x)) = 0$, because $f(f(-8)) = f(-4) = 0$. Find all the solutions of the equation $f(f(f(x))) = 0$.
A -4; 0 **B** -8; -4; 0 **C** -12; -8; -4; 0 **D** -16; -12; -8; -4; 0 **E** No solutions

5-POINT QUESTIONS

J21. What is the ratio of the areas of the triangles ADE and ABC in the picture?
A $\frac{9}{4}$ **B** $\frac{7}{3}$ **C** $\frac{4}{5}$ **D** $\frac{15}{10}$ **E** $\frac{26}{9}$

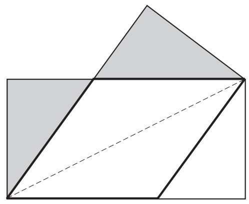


J22. The rectangle $ABCD$ has area 36. A circle with center in point O is inscribed in the triangle ABD. What is the area of the rectangle $OPCR$?
A 24 **B** 6π **C** 18 **D** $12\sqrt{2}$
E It depends on the ratio of the sides AB and AD



J23. The children K, L, M and N made the following assertions:
K: L, M and N are girls; L: K, M and N are boys;
M: K and L are lying; N: K, L and M are telling the truth.
How many of the children were telling the truth?
A 0 **B** 1 **C** 2 **D** 3 **E** Impossible to determine

J24. A rectangular sheet of paper with measures 6×12 is folded along its diagonal. The shaded parts sticking out over the edge of the overlapping area are cut off and the sheet is unfolded. Now it has the shape of a rhombus. Find the length of the side of the rhombus.
A $\frac{7\sqrt{5}}{2}$ **B** 7.35 **C** 7.5 **D** 7.85 **E** 8.1



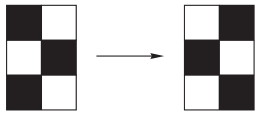
J25. How many distinct pairs $(x; y)$ satisfy the equation $(x + y)^2 = xy$?
A 0 **B** 1 **C** 2 **D** 3 **E** Infinitely many

J26. What is the greatest number of consecutive integers such that the sum of the digits of none of them is divisible by 5?
A 5 **B** 6 **C** 7 **D** 8 **E** 9

J27. On a bookshelf there are 50 math and physics books. No two physics books stand side by side, but every math book has a math neighbour. Which of the following statements *may turn out to be false*?
A The number of math books is at least 32
B The number of physics books is at most 17
C There are 3 math books standing in succession
D If the number of physics books is 17, then at least one of them is the first or the last on the bookshelf
E Among any 9 successive books, at least 6 are math books

J28. We take three different numbers from the numbers 1, 4, 7, 10, 13, 16, 19, 22, 25, 28 and find their sum. How many different sums can we obtain?
A 13 **B** 21 **C** 22 **D** 30 **E** 120

J29. Unit squares of a squared board 2×3 are coloured black and white like a chessboard (see picture). Determine the minimum number of steps necessary to achieve the reverse of the left board, following the rule: in each step, we must repaint two unit squares that have a joint edge, but we must repaint a black square with green, a green square with white and a white square with black.



A 3 **B** 5 **C** 6 **D** 8 **E** 9

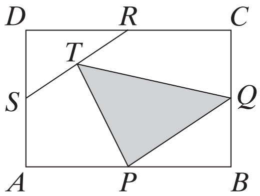
J30. We wrote down all the integers of 1 to 5 digits we could, using only the two digits 0 and 1. How many 1's did we write?
A 36 **B** 48 **C** 80 **D** 160 **E** 320

STUDENT (grades 11 and 12)

3-POINT QUESTIONS

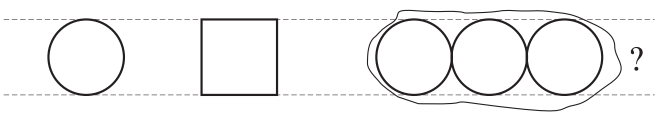
S1. Ann has a box containing 9 pencils. At least one of them is blue. Among every 4 of the pencils at least two have the same colour, and among every 5 of the pencils at most three have the same colour. What is the number of blue pencils?
A 2 **B** 3 **C** 4 **D** 1 **E** Impossible to determine

S2. In a rectangle $ABCD$, let P, Q, R and S be the midpoints of sides AB, BC, CD and AD, respectively, and let T be the midpoint of segment RS. Which fraction of the area of $ABCD$ does triangle PQT cover?



A $\frac{5}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{5}$ **D** $\frac{1}{6}$ **E** $\frac{3}{8}$

S3. The area of the wooden square equals a. The area of each wooden circle equals b. Three circles are lined up as shown in the picture. If we tie together the three circles with a thread as short as possible, without moving them, what is the area inside the thread?



A $3b$ **B** $2a + b$ **C** $a + 2b$ **D** $3a$ **E** $a + b$

S4. Alan was calculating the volume of a sphere, but in the calculation he mistakenly used the value of the diameter instead of the radius of the sphere. What should he do with his result to get the correct answer?
A Divide it by 2 **B** Divide it by 4 **C** Multiply it by 6 **D** Divide it by 8
E Multiply it by 8

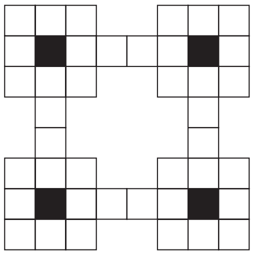
S5. If n is a positive integer, then $2^{n+2003} + 2^{n+2003}$ is equal to
A 2^{n+2004} **B** $2^{2n+4006}$ **C** $4^{2n+4006}$ **D** $4^{2n+2003}$ **E** 4^{n+2003}

S6. For which of the following settings does a triangle ABC exist?
A $AB = 11\,\text{cm}$, $BC = 19\,\text{cm}$, $CA = 7\,\text{cm}$
B $AB = 11\,\text{cm}$, $BC = 7\,\text{cm}$, $\angle BAC = 60^\circ$
C $AB = 11\,\text{cm}$, $CA = 7\,\text{cm}$, $\angle CBA = 128^\circ$
D $AB = 11\,\text{cm}$, $\angle BAC = 60^\circ$, $\angle CBA = 128^\circ$
E For none of them

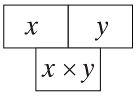
S7. The average number of students accepted by a school in the four years 1998–2001 was 325 students per year. The average number of students accepted by the school in the five years 1998–2002 is 20% higher. How many students did this school accept in 2002?
A 650 **B** 600 **C** 455 **D** 390 **E** 345

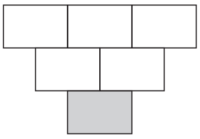
S8. Find all values of the parameter m for which the curves $x^2 + y^2 = 1$ and $y = x^2 + m$ have exactly one common point.
A $-\frac{5}{4}; -1; 1$ **B** $-\frac{5}{4}; 1$ **C** $-1; 1$ **D** $-\frac{5}{4}$ **E** 1

S9. The composite board shown in the picture consists of 44 fields 1×1. How many possibilities are there to cover all 40 white fields with 20 rectangular stones 1×2? (The board cannot be turned. Two possibilities are different if at least one stone lies in another way.)
A 8 **B** 16 **C** 32 **D** 64 **E** 100



S10. According to the rule given in the left picture below, we construct a numerical triangle with an integer number greater than 1 in each cell. Which of the numbers given in the answers cannot appear in the shaded cell?

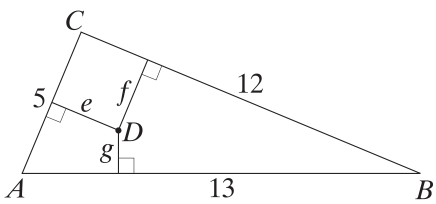




A 154 **B** 100 **C** 90 **D** 88 **E** 60

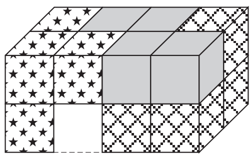
4-POINT QUESTIONS

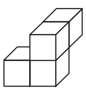
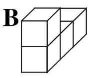
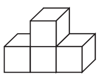
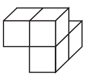
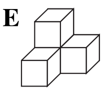
S11. Let ABC be a triangle with area 30. Let D be any point in its interior and let e, f and g denote the distances from D to the sides of the triangle.



What is the value of the expression $5e + 12f + 13g$?
A 120 **B** 90 **C** 60 **D** 30
E Impossible to find the value without knowing the exact location of D

S12. A rectangular parallelepiped was composed of 4 pieces, each consisting of 4 little cubes. Then one piece was removed (see picture). Which one?



A  **B**  **C**  **D**  **E** 

S13. Two white and eight gray seagulls were flying over a river. Suddenly, they all randomly sat down at the bank forming a line. What is the probability that the two white seagulls were sitting side by side?
A $\frac{1}{5}$ **B** $\frac{1}{6}$ **C** $\frac{1}{7}$ **D** $\frac{1}{8}$ **E** $\frac{1}{9}$

S14. The value of

$$\sqrt{1+2000\sqrt{1+2001\sqrt{1+2002\sqrt{1+2003\cdot 2005}}}}$$

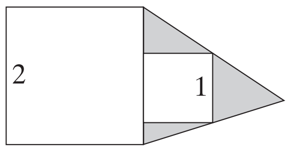
is equal to
A 2000 **B** 2001 **C** 2002 **D** 2003 **E** 2004

S15. Numbers 12, 13 and 15 are the lengths (perhaps not in order) of two sides of an acute-angled triangle and of the height over the third side of this triangle. Find the area of the triangle.
A 168 **B** 80 **C** 84 **D** $6\sqrt{65}$ **E** Impossible to find

S16. The sequence $1^7, 2^7, 3^7, \ldots$ is constructed of the seventh powers of all positive integers. How many terms of this sequence lie between the numbers 5^{21} and 2^{49}?
A 13 **B** 8 **C** 5 **D** 3 **E** 2

S17. We know that $10^n + 1$ is a multiple of 101, and n is a 2-digit number. What is the largest possible value of n?
A 92 **B** 94 **C** 96 **D** 98 **E** 99

S18. The diagram shows two squares: one has a side with a length of 2 and the other (abut on the first square) has a side with a length of 1. What is the area of the shaded zone?
A 1 **B** 2 **C** $2\sqrt{2}$ **D** 4
E It depends on the position of the smaller square

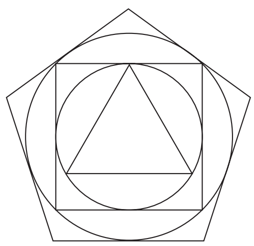


S19. How many of the functions $f(x) = 0,\ f(x) = \frac{1}{2},\ f(x) = 1,\ f(x) = x,\ f(x) = -x$ satisfy the equation $f(x^2 + y^2) = f^2(x) + f^2(y)$?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

S20. If $a^4 + \frac{1}{a^4} = 4$, then $a^6 + \frac{1}{a^6}$ is equal to
A $4\sqrt{6}$ **B** $3\sqrt{6}$ **C** 6 **D** $5\sqrt{6}$ **E** $6\sqrt{6}$

5-POINT QUESTIONS

S21. We first draw an equilateral triangle, then draw the circumcircle of this triangle, then circumscribe a square to this circle. After drawing another circumcircle, we circumscribe a regular pentagon to this circle, and so on. We repeat this construction with new circles and new regular polygons (each with one side more than the preceding one) until we draw a 16-sided regular polygon. How many disjoint regions are there inside the last polygon?
A 232 **B** 240 **C** 248 **D** 264 **E** 272



S22. A point $P(x; y)$ lies on a circle with center $M(2; 2)$ and radius r. We know that $y = r > 2$ and x, y and r are all positive integers. What is the smallest possible value of x?
A 2 **B** 4 **C** 6 **D** 8 **E** 10

S23. The four positive integers A, B, $A - B$, $A + B$ are all prime. Then the sum of them
A is even **B** is a multiple of 3 **C** is a multiple of 5 **D** is a multiple of 7 **E** is prime

S24. A manager in a store has to determine the price of a sweater. Market research gives him the following information: If the price is \$75 then 100 teens will buy the sweaters. The price can be increased or decreased several times by units of \$5. Each time the price is increased by \$5, 20 fewer teens will buy the sweaters. However, each time the price is decreased by \$5, 20 sweaters more will be sold. The sweater costs the company \$30 apiece. What is the sale price that maximizes profits?
A 85 **B** 80 **C** 75 **D** 70 **E** 65

S25. How many distinct pairs $(x; y)$ satisfy the equation $(x + y)^2 = (x + 3)(y - 3)$?
A 0 **B** 1 **C** 2 **D** 3 **E** Infinitely many

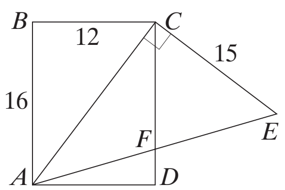
S26. A sequence $a_0, a_1, a_2, \ldots$ is defined in the following way:

$$a_0 = 4, \quad a_1 = 6, \quad a_{n+1} = \frac{a_n}{a_{n-1}} \qquad (n \geqslant 1).$$

Then a_{2003} is equal to

A $\frac{3}{2}$ **B** $\frac{2}{3}$ **C** 4 **D** $\frac{1}{4}$ **E** $\frac{1}{6}$

S27. In the picture $ABCD$ is a rectangle with $AB = 16$, $BC = 12$. Let E be such a point that $AC \perp CE$, $CE = 15$.



If F is the point of intersection of segments AE and CD, then the area of the triangle ACF is equal to
A 75 **B** 80 **C** 96 **D** 72 **E** 48

S28. We can put an arrow on one end of the edge of a cube, defining a vector, and put an arrow on the other end of the edge, defining the opposite vector. We put an arrow on each edge and then add up all 12 vectors obtained. How many different values of sum of vectors can we obtain in this way?
A 25 **B** 27 **C** 64 **D** 100 **E** 125

S29. We are given the 6 vertices of a regular hexagon and all line segments joining any two of these points. We call two such segments *strangers* if they have no common point (including end points). How many pairs of strangers are there?
A 26 **B** 28 **C** 30 **D** 34 **E** 36

S30. Let f be a polynomial such that $f(x^2 + 1) = x^4 + 4x^2$. Determine $f(x^2 - 1)$.
A $x^4 - 4x^2$ **B** x^4 **C** $x^4 + 4x^2 - 4$ **D** $x^4 - 4$ **E** Another answer