The 17^{th} Romanian Master of Mathematics Competition

Day 1: The 25^{th} of February, 2026, Bucharest

Language: English

Problem 1. Let n be a positive integer. Alice draws a unit area triangle on the board. Then she draws additional triangles by performing n moves in a row. On each move, she chooses a drawn triangle Δ with no marked points in its interior, marks a point P in its interior, and draws three smaller triangles by joining P to each vertex of Δ with a segment.

Once these n moves have been performed, Bob chooses three distinct drawn triangles Δ_1, Δ_2, and Δ_3 which contain no marked points in their interiors, such that Δ_2 shares one side with Δ_1 and another with Δ_3. In terms of n, determine the largest possible constant c such that Bob can guarantee that the sum of the areas of Δ_1, Δ_2, and Δ_3 is at least c, regardless of Alice's choices.

Problem 2. Let $p \geq 11$ be a prime. Suppose that, if a and b are integers such that $1 \leq a < b \leq p-3$, then $b! - a!$ is not divisible by p. Prove that $p - 5$ is divisible by 8.

Problem 3. Let $\mathcal{S}$ be a finite subset of $\mathbb{R}^3$. Prove that there exist three polynomials $P(x, y, z)$, $Q(x, y, z)$ and $R(x, y, z)$ with real coefficients, such that a triple of real numbers (a, b, c) is in $\mathcal{S}$ if and only if the system of equations

$$P(x, y, z) = a,$$

$$Q(x, y, z) = b,$$

$$R(x, y, z) = c,$$

does **not** have a solution in real numbers x, y, and z.

Each problem is worth 7 marks.

Time allowed: $4\frac{1}{2}$ hours.

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Day 2: The 26th of February, 2026, Bucharest

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Problem 4. For any positive integer m, let $\varphi(m)$ be the number of positive integers less than or equal to m and coprime to m. Define $\varphi_0(m) = m$ and, for each positive integer k, $\varphi_k(m) = \varphi(\varphi_{k-1}(m))$. For any integer $n \geq 3$, prove that

$$\varphi_0(2^n - 3) \cdot \varphi_1(2^n - 3) \cdot \varphi_2(2^n - 3) \cdot \ldots \cdot \varphi_n(2^n - 3)$$

has at most n distinct prime divisors.

Problem 5. Let ABC be a triangle with $AB < AC$, let O be its circumcentre and let $XYZT$ be a parallelogram inside triangle ABC such that

$$\angle AXB = \angle AZC, \ \angle AZB = \angle AXC,$$

$$\angle AYB = \angle ATC, \ \angle ATB = \angle AYC.$$

Prove that the diagonals XZ and YT of the parallelogram intersect on the circumcircle of BOC.

Problem 6. Let $k > 1$ be an integer, and let S denote the set of all $(k+1)$-tuples of integers $X = (x_1, \ldots, x_{k+1})$ such that $1 \leq x_1 < \cdots < x_{k+1} \leq k^2+1$. If σ is a permutation of the numbers $1, 2, \ldots, k^2 + 1$, say that an element X of S is *σ-nice* if the sequence $\sigma(x_1), \sigma(x_2), \ldots, \sigma(x_{k+1})$ is monotone. Prove that

$$\min_{1 \leq i \leq k} \left\lfloor \frac{x_i}{i} \right\rfloor + \min_{2 \leq i \leq k+1} \left\lfloor \frac{k^2 + 2 - x_i}{k + 2 - i} \right\rfloor \geq k + 1$$

if and only if there exists a permutation σ such that X is the unique σ-nice tuple in S.

Each problem is worth 7 marks.
Time allowed: $4\frac{1}{2}$ hours.