Algebra	
A1	Prove that for all sufficiently large positive integers d, at least 99% of the polynomials of the form $$\sum_{i\leqslant d}\sum_{j\leqslant d}\pm x^i y^j$$ are irreducible over the integers.
A2	Let $n>1$ be a positive integer and $\mathcal{S}$ be the set of n^{th} roots of unity. Suppose P is an n-variable polynomial with complex coefficients such that for all $a_1,\dots,a_n\in\mathcal{S}$, $P(a_1,\dots,a_n)=0$ if and only if $a_1,\dots,a_n$ are all different. What is the smallest possible degree of P? *Adam Ardeishar and Michael Ren*

Combinatorics	
C1	Bethan is playing a game on an $n\times n$ grid consisting of n^2 cells. A move consists of placing a counter in an unoccupied cell C where the $2n-2$ other cells in the same row or column as C contain an even number of counters. After making M moves Bethan realises she cannot make any more moves. Determine the minimum value of M. *United Kingdom, Sam Bealing*
C2	Let n be a positive integer, and let $\mathcal{C}$ be a collection of subsets of $\{1,2,\dots,2^n\}$ satisfying both of the following conditions: 1. Every (2^n-1)-element subset of $\{1,2,\dots,2^n\}$ is a member of $\mathcal{C}$, and 2. Every non-empty member C of $\mathcal{C}$ contains an element c such that $C\setminus\{c\}$ is again a member of $\mathcal{C}$. Determine the smallest size $\mathcal{C}$ may have. *Serbia, Pavle Martinović*
C3	Determine the smallest positive integer k satisfying the following condition: For any configuration of chess queens on a 100×100 chequered board, the queens can be coloured one of k colours so that no two queens of the same colour attack each other. *Russia, Sergei Avgustinovich and Dmitry Khramtsov*
C4	A ternary sequence is one whose terms all lie in the set $\{0,1,2\}$. Let w be a length n ternary sequence $(a_1,\dots,a_n)$. Prove that w can be extended leftwards and rightwards to a length $m=6n$ ternary sequence $$(d_1,\dots,d_m)=(b_1,\dots,b_p,a_1,\dots,a_n,c_1,\dots,c_q),\quad p,q\geqslant 0,$$ containing no length $t>2n$ palindromic subsequence. (A sequence is called palindromic if it reads the same rightwards and leftwards. A length t subsequence of $(d_1,\dots,d_m)$ is a sequence of the form $(d_{i_1},\dots,d_{i_t})$, where $1\leqslant i_1<\dots<i_t\leqslant m$.)

Geometry	
G1	The incircle of a scalene triangle ABC touches the sides BC, CA, and AB at points D, E, and F, respectively. Triangles APE and AQF are constructed outside the triangle so that $$AP = PE, AQ = QF, \angle APE = \angle ACB, \text{ and } \angle AQF = \angle ABC.$$ Let M be the midpoint of BC. Find $\angle QMP$ in terms of the angles of the triangle ABC. *Iran, Shayan Talaei*
G2	Let ABC be an acute scalene triangle, and let A_1, B_1, C_1 be the feet of the altitudes from A, B, C. Let A_2 be the intersection of the tangents to the circle ABC at B, C and define B_2, C_2 similarly. Let A_2A_1 intersect the circle $A_2B_2C_2$ again at A_3 and define B_3, C_3 similarly. Show that the circles AA_1A_3, BB_1B_3, and CC_1C_3 all have two common points, X_1 and X_2 which both lie on the Euler line of the triangle ABC. *United Kingdom, Joe Benton*
G3	In the triangle ABC with circumcircle Γ, the incircle ω touches sides BC, CA, and AB at D, E, and F, respectively. The line through D perpendicular to EF meets ω at $K \neq D$. Line AK meets Γ at $L \neq A$. Rays KI and IL meet the circumcircle of triangle BIC at $Q \neq I$ and $P \neq I$, respectively. The circumcircles of triangles KFB and KEC meet EF at $R \neq F$ and $S \neq E$, respectively. Prove that $PQRS$ is cyclic. *India, Anant Mugdal*

Number Theory	
N1	Determine all pairs of positive integers (m, n) for which there exists a bijective function $$f: \mathbb{Z}_m \times \mathbb{Z}_n \to \mathbb{Z}_m \times \mathbb{Z}_n$$ such that the vectors $f(\mathbf{v}) + \mathbf{v}$, as $\mathbf{v}$ runs through all of $\mathbb{Z}_m \times \mathbb{Z}_n$, are pairwise distinct. (For any integers a and b, the vectors $[a, b]$, $[a+m, b]$ and $[a, b+n]$ are treated as equal.) *Poland, Wojciech Nadara*
N2	For a positive integer n, let $\varphi(n)$ and $d(n)$ denote the value of the Euler phi function at n and the number of positive divisors of n, respectively. Prove that there are infinitely many positive integers n such that $\varphi(n)$ and $d(n)$ are both perfect squares. *Finland, Olli Järviniemi*