

(100-170):

1.

2.  $ABCD$ ,  $\angle BAD = \alpha$   $\angle BCD = \beta$ .

$$AC^2 \cdot BD^2 = AB^2 \cdot CD^2 + BC^2 \cdot DA^2 - 2AB \cdot BC \cdot CD \cdot DA \cos(\alpha + \beta).$$

$$\cos(\alpha + \beta) \geq -1,$$

$$-2AB \cdot BC \cdot CD \cdot DA \cdot \cos(\alpha + \beta) \leq 2AB \cdot BC \cdot CD \cdot DA$$

2

$$AC^2 \cdot BD^2 \leq AB^2 \cdot CD^2 + BC^2 \cdot DA^2 + 2AB \cdot BC \cdot CD \cdot DA = (AB \cdot CD + BC \cdot DA)^2.$$

$$AC \cdot BD \leq AB \cdot CD + BC \cdot DA.$$

$$AC \cdot BD = AB \cdot CD + BC \cdot DA$$

$$\cos(\alpha + \beta) = -1, \dots \alpha + \beta = 180^\circ.$$

$$\alpha + \beta$$

360°.

2.

K

ABCD

$$\angle KDA = \angle BAC \quad \angle KAD = \angle ACB \quad (\dots).$$

$$ABCD \quad \angle MAB = \angle ACD, \quad \angle MBA = \angle CAD. \quad \triangle UADK \sim \triangle CAB$$

$$AK = \frac{BC \cdot DA}{AC} \quad DK = \frac{AB \cdot DA}{AC}, \quad \triangle ABM \sim \triangle CAD$$

$$AM = \frac{CD \cdot AB}{AC} \quad BM = \frac{DA \cdot AB}{AC}. \quad DK = BM.$$

$$\angle KDA + \angle ADB + \angle DBA + \angle ABM = \angle ADB + \angle DBA + \angle BAD = 180^\circ$$

( $\triangle ABD$ ).

$$\angle KDB + \angle DBM = 180^\circ$$

$DK \parallel BM$

$$DK = BM$$

$KMBD$

$$KM = BM.$$

AMK.

$$\begin{aligned} BD^2 &= KM^2 = AK^2 + AM^2 - 2AK \cdot AM \cos(\alpha + \beta) \\ &= \frac{BC^2 \cdot DA^2}{AC^2} + \frac{CD^2 \cdot AB^2}{AC^2} - 2 \frac{BC \cdot DA \cdot CD \cdot AB}{AC^2} \cos(\alpha + \beta) \end{aligned}$$

$$AC^2 \cdot BD^2 = AB^2 \cdot CD^2 + BC^2 \cdot DA^2 - 2AB \cdot BC \cdot CD \cdot DA \cos(\alpha + \beta). \quad \blacklozenge$$

2.

1.

ABCD,

AC BD  
, B , C D

C

. B D

$$\frac{AO}{CO}$$

$$\frac{AO}{CO} = \frac{m}{n},$$

m n

$$\frac{AO}{AO+CO} = \frac{m}{m+n}$$

$$AO = \frac{m}{m+n} AC$$

C

(

$$AH_1 \perp BD \quad (H_1 \in BD) \quad CH_2 \perp BD \quad (H_2 \in BD). \quad H_1 \sim C \quad H_2,$$

$$\frac{AO}{CO} = \frac{AH_1}{CH_2}.$$

$$\frac{P_{ABD}}{P_{BDC}} = \frac{BD \cdot AH_1}{BD \cdot CH_2} = \frac{AH_1}{CH_2}.$$

$$\frac{AO}{CO} = \frac{S_{ABD}}{S_{BDC}} = \frac{AB \cdot DA \sin \angle BAD}{BC \cdot CD \sin \angle BCD} = r \frac{\sin \angle BAD}{\sin \angle BCD},$$

$$r = \frac{AB \cdot DA}{BC \cdot CD}$$

$$\frac{AO}{CO} = r \frac{\sin \angle BAD}{\sin \angle BCD} = r \frac{\sin^2 \angle BAD}{\sin \angle BCD \cdot \sin \angle BAD} = r \frac{1 - \cos^2 \angle BAD}{\sin \angle BCD \cdot \sin \angle BAD}.$$

BD

$$\cos \angle BAD = \frac{AB^2 + DA^2 - BD^2}{2AB \cdot DA} = q,$$

q

$$\cos^2 \angle BAD$$

$$1 - \cos^2 \angle BAD = 1 - q^2$$

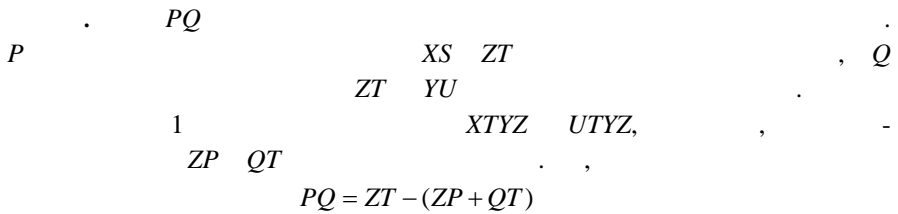
$$\frac{AO}{CO} = \frac{r(1-q^2)}{\sin \angle BCD \cdot \sin \angle BAD} \cdot \frac{ABCD}{ABCD}, \quad (2)$$

$$\cos(\angle BAD + \angle BCD) = \frac{AB^2 \cdot CD^2 + BC^2 \cdot DA^2 - AC^2 \cdot BD^2}{2AB \cdot BC \cdot CD \cdot DA} = s,$$

$$s = \cos(\angle BAD + \angle BCD) = \cos \angle BAD \cdot \cos \angle BCD - \sin \angle BAD \cdot \sin \angle BCD.$$

$$\sin \angle BAD \cdot \sin \angle BCD = \cos \angle BAD \cdot \cos \angle BCD - s = \frac{ABD}{ABD} - \frac{BCD}{BCD} - s = \frac{AO}{CO}.$$

2. ( , 23 2003 )



1. , ∴ , , , 1986 ( )