JUNIOR (grades 9 and 10)

3-POINT QUESTIONS

J1. The value of the expression $(1-2)-(3-4)-(5-6)-\cdots-(99-100)$ is equal to
A -50 **B** 49 **C** -48 **D** 48 **E** 50

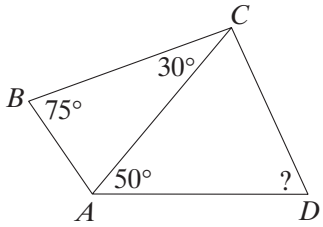
J2. Edward has 2004 marbles. Half of them are blue, one quarter are red, and one sixth are green. How many marbles are of some other color?
A 167 **B** 334 **C** 501 **D** 1001 **E** 1837

J3. A pyramid has 7 faces. How many edges does it have?
A 7 **B** 9 **C** 12 **D** 14 **E** 21

J4. The ground plan of a building has a rectangular shape with parameters of $40\,\text{m} \times 60\,\text{m}$. In the diagram the ground plan of the building has a perimeter of 100 cm. What is the scale of the diagram?
A 1:100 **B** 1:150 **C** 1:160 **D** 1:170 **E** 1:200

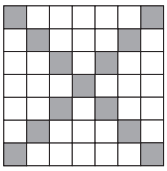
J5. Tom and Ron both had some one-euro coins. When Tom got 5 more coins from his grandfather, he had twice as many coins as Ron. And if Tom now gave 12 coins to his grandmother, he would have half as many coins as Ron. How many coins did Tom have at the very beginning?
A 5 **B** 7 **C** 9 **D** 11 **E** 45

J6. Some angles in the quadrilateral $ABCD$ are shown in the figure. If $BC = AD$, then what is the angle ADC?
A 30° **B** 50° **C** 55° **D** 65° **E** 70°



J7. There are some boletuses and orange-caps in a basket – 30 mushrooms altogether. If we randomly take out 12 mushrooms, there will be at least one orange-cap among them. If we randomly take out 20 mushrooms, there will be at least one boletus among them. How many boletuses are there in the basket?
A 11 **B** 12 **C** 19 **D** 20 **E** 21

J8. In a square 2003×2003, the squares 1×1 on the diagonals are colored (like in the picture, where the square is 7×7). How many white squares are there?
A 2002^2 **B** 2002×2001 **C** 2001^2 **D** 2003×2002
E $2003^2 - 2004$



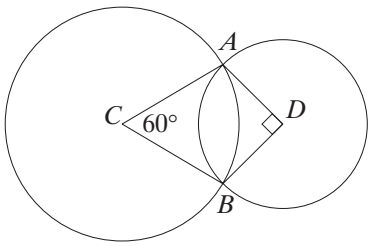
J9. The dartboard shown consists of an inner black circle and 2 rings around it. The width of each ring is equal to the radius of the black circle. How many times greater is the area of the grey ring than the area of the inner black circle?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

J10. After gathering 770 nuts, three girls divided them in proportion to their ages. For every 3 nuts Oxana took, Ira took 4. For every 7 nuts Natalya took, Ira took 6. How many nuts did the youngest girl get?
A 264 **B** 256 **C** 218 **D** 198 **E** 180

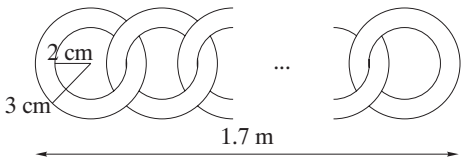
4-POINT QUESTIONS

J11. Each of five children thinks of a number, which can be either 1, 2, or 4. Their numbers are multiplied. Which number could be the result?
A 100 **B** 120 **C** 256 **D** 768 **E** 2048

J12. The circles with centers C and D meet at the points A and B, as shown. Angle $ACB = 60°$ and angle $ADB = 90°$. How many times longer is the radius of the larger circle than the radius of the smaller circle?
A $\frac{4}{3}$ **B** $\sqrt{2}$ **C** $\frac{3}{2}$ **D** $\sqrt{3}$ **E** 2

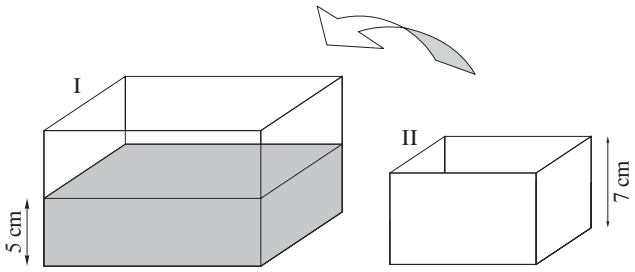


J13. We link rings together as shown in the figure below; the length of the chain is 1.7 m.



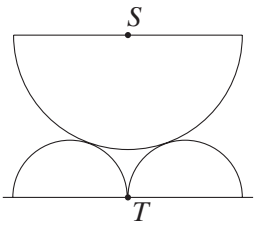
How many rings are there?
A 17 **B** 21 **C** 30 **D** 42 **E** 85

J14. In tank I, whose base has an area of $2\,\text{dm}^2$ and whose height is 10 cm, the water is 5 cm high. An empty tank II with a base of area $1\,\text{dm}^2$ and a height of 7 cm is placed in tank I. The water of tank I rises, of course, and spills over into tank II.
What level does the water reach in tank II?
A 1 cm **B** 2 cm **C** 3 cm
D 4 cm **E** 5 cm



J15. The hour hand of a clock is 4 cm long, and the minute hand is 8 cm long. What is the ratio of the distances travelled by the tips of the two hands between 2 pm and 5 pm?
A 1:2 **B** 1:4 **C** 1:6 **D** 1:12 **E** 1:24

J16. Three semi-circles, the diameters of two of which are equal to 4 and of the third to 8, are arranged as seen in the picture. What is the distance from the center S of the bigger semi-circle to the tangent point T of the smaller semi-circles?
A 6 **B** $\sqrt{32}$ **C** 5.7 **D** $\sqrt{40}$ **E** 5



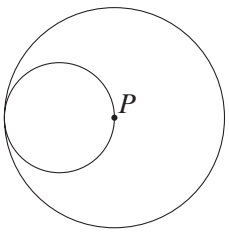
J17. A quiz has twenty questions with seven points awarded for each correct answer, two points deducted for each wrong answer, and zero for each question omitted. Andrew scores 87 points. How many questions did he omit?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

J18. Caroline wants to write the numbers 1, 2, 3, 4 in the square 4×4 in such a way that every row and every column has each of the numbers. You see how she started. In how many different ways can she finish?
A 1 **B** 2 **C** 4 **D** 16 **E** 128

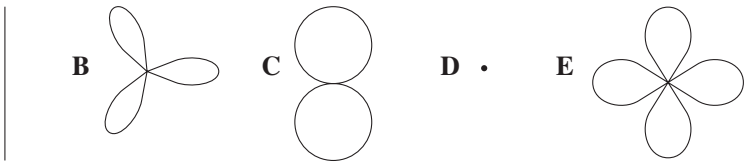
1			
2	1		
	3		
	4		

J19. How many numbers exist between 100 and 200 which can have only the prime factors 2 and 3?
A 1 **B** 3 **C** 4 **D** 5 **E** 6

J20. The diagram shows two tangential circles with radii in the ratio 1: 2. The smaller circle rolls around the inside of the large circle. Which of the following is the path traced out by the point P of the smaller circle?

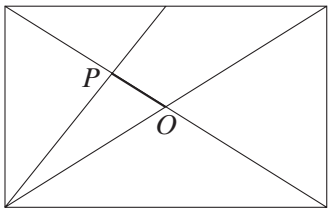


A **B** **C** **D** **E**



5-POINT QUESTIONS

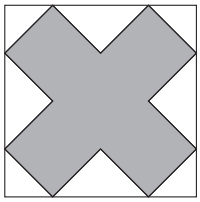
J21. In a rectangle we draw both diagonals and the segment which joins a vertex with the midpoint of one of the sides, as shown in the picture. What is the result of dividing the length of the diagonal by the length of segment OP?



A 3 **B** 6 **C** $\frac{13}{3}$ **D** 4
E It depends on the dimensions of the rectangle

J22. The real numbers a and b have different signs. Which of the numbers given below is the largest one?
A $|a^2 - b^2|$ **B** $(|a| - |b|)^2$ **C** $(a - b)^2$ **D** $(a + b)^2$ **E** $a^2 + b^2$

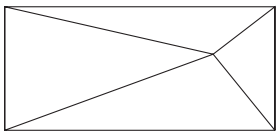
J23. The diagram shows a square and an equilateral right-angled cross-shaped dodecagon. The length of the perimeter of the dodecagon is 36 cm. What, in cm^2, is the area of the square?



A 48 **B** 72 **C** 108 **D** $36\sqrt{2}$ **E** 144

J24. How many 3-digit numbers smaller than 200 have the property that the number $(n + 1)(n + 2)(n + 3)$ is divisible by 7?
A 42 **B** 38 **C** 34 **D** 28 **E** 16

J25. A rectangle is divided into 4 triangles as shown in the figure. Of the following possibilities for the areas of the triangles at most one can be true. Which one is it?



A 4, 5, 8, 9 **B** 3, 5, 6, 7 **C** 5, 6, 7, 12
D 10, 11, 12, 19 **E** 5, 6, 8, 10

J26. This is a multiplication table. Which two letters represent the same number?

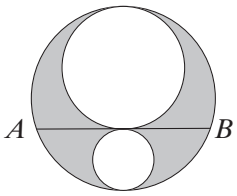
×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

A L and M **B** P and N **C** R and S
D K and R **E** M and T

J27. After one operation, the triplet (a, b, c) turns into triplet $(b + c, c + a, a + b)$. After 2004 successive operations, the triplet $(1, 3, 5)$ turned into a triplet (x, y, z). What is the difference $x - y$ equal to?
A -2 **B** 2 **C** 4008 **D** 2004 **E** $(-2)^{2004}$

J28. How many 8-digit numbers $\overline{a_1a_2a_3a_4a_5a_6a_7a_8}$ whose digits can only be 0s or 1s ($a_1 = 1$) have the property $a_1 + a_3 + a_5 + a_7 = a_2 + a_4 + a_6 + a_8$?
A 2^7 **B** 35 **C** 49 **D** 16 **E** 32

J29. The area of the shaded shape is equal to 2π (see the picture). What is the value of the chord AB?
A 1 **B** 2 **C** 3 **D** 4 **E** It's impossible to determine.



J30. All the integers from 1 to 10,000 were written down on a blackboard. After that the numbers that are not divisible by 5 or 11 were erased. Then the 2004th element of the sequence obtained was:
A 1000 **B** 5000 **C** 10,000 **D** 6545 **E** 7348

STUDENT (grades 11 and 12)

3-POINT QUESTIONS

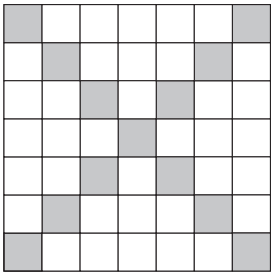
S1. If m pens are bought at n euros each, and n pens at m euros each ($m \neq n$), then the average cost per pen, in euros, is:
A 1 **B** $\frac{m+n}{2}$ **C** $\frac{2mn}{m+n}$ **D** mn **E** $\sqrt{mn}$

S2. A pyramid has 17 faces. How many vertices does it have?
A 16 **B** 17 **C** 18 **D** 32 **E** 34

S3. The smallest real number satisfying the inequality $x^2 - 2004 \leqslant 0$ is:
A -2004 **B** 2004 **C** 0 **D** $\sqrt{2004}$ **E** $-\sqrt{2004}$

S4. Each Martian has one, two, or three tentacles on its head. Exactly 1% of the Martian population consists of individuals with three tentacles, exactly 97% comprise Martians with two tentacles, and the remaining 2% consists of individuals with one tentacle. What percent of Martians have more tentacles on their head than the average of the whole Martian population?
A 1% **B** 3% **C** 97% **D** 98% **E** 99%

S5. In a square of side s, where s is an odd integer, the squares of side 1 on the diagonals are colored (like in the picture, where the square is of side 7). How many white squares are there?
A s^2+1-2s **B** s^2+4-4s **C** $2s^2+1-4s$ **D** s^2-1-2s **E** s^2-2s



S6. How many two-digit numbers exist whose square and cube end in the same digit?
A 1 **B** 9 **C** 10 **D** 21 **E** More than 30

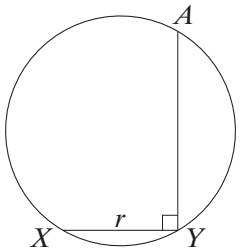
S7. Square $ABCD$ consists of 18 smaller squares, 17 of which have sides equal to 1. The area of the square $ABCD$ is:
A 25 **B** 49 **C** 81 **D** 100 **E** 225

S8. How many right triangles can be formed by joining three vertices of a given regular 14-gon?
A 72 **B** 82 **C** 84 **D** 88 **E** Other answer

S9. This is a multiplication table. What two letters represent the same number?
A L and M **B** P and N **C** R and S **D** K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

S10. On the circumference of radius r three points X, Y and A are marked such that $XY = r$, $XY \perp AY$ (see the figure).
How many degrees has the angle XAY?
A $22\frac{1}{2}$ **B** 30 **C** 45 **D** 60 **E** $67\frac{1}{2}$

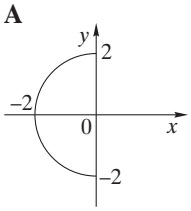


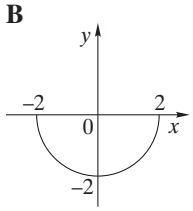
4-POINT QUESTIONS

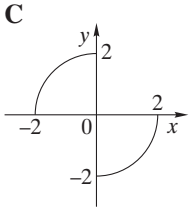
S11. In the plane Oxy, how many squares with vertex $A(-1; -1)$ exist such that at least one of the coordinate axes is an axis of symmetry of the square?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

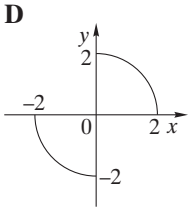
S12. There are 100 cards in a non-transparent envelope, numbered with integers from 1 to 100. There is a different number on each card. What is the smallest number of cards we have to pull out of the envelope at random to be sure that the product of the numbers on the chosen cards is divisible by 4?
A 4 **B** 52 **C** 50 **D** 48 **E** 96

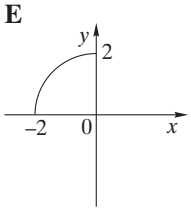
S13. The set of all pairs (x, y) which satisfy conditions $xy \leqslant 0$ and $x^2 + y^2 = 4$ is on the graph:

A


B


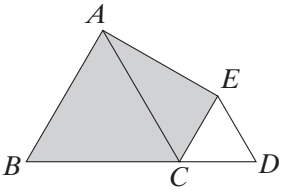
C


D


E


S14. In the figure the two equilateral triangles ABC and ECD have sides of length 2 and 1 respectively. The area of the quadrilateral $ABCE$ is:

A $\frac{5\sqrt{3}}{3}$ **B** $\frac{4+5\sqrt{3}}{5}$ **C** 3 **D** $\frac{6+\sqrt{3}}{4}$ **E** $\frac{3\sqrt{3}}{2}$

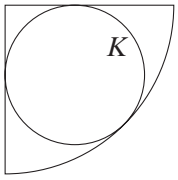


S15. How many positive integers can be written as $a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 + a_3 \cdot 3^3 + a_4 \cdot 3^4$ if a_0, a_1, a_2, a_3, a_4 belong to the set $\{-1, 0, 1\}$?
A 5 **B** 80 **C** 81 **D** 121 **E** 243

S16. The number $(\sqrt{22 + 12\sqrt{2}} - \sqrt{22 - 12\sqrt{2}})^2$ is
A negative **B** equal to zero **C** a fourth power of a non-zero integer
D equal to $11\sqrt{2}$ **E** a positive integer divisible by 5

S17. How many vertices are there in a regular polygon the sum of whose interior angles is one seventh of that of a regular 16-gon?
A 3 **B** 4 **C** 6 **D** 7 **E** 10

S18. A circle K is inscribed in a quarter circle with radius 6 as shown in the figure. What is the radius of circle K?
A $\frac{6-\sqrt{2}}{2}$ **B** $\frac{3\sqrt{2}}{2}$ **C** 2.5 **D** 3 **E** $6(\sqrt{2} - 1)$



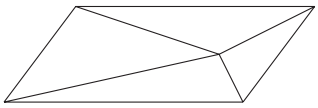
S19. For a geometric sequence (a_n) the following inequalities hold: $a_3 < a_2 < a_4$. Then
A $a_3a_4 > 0$ **B** $a_2a_3 < 0$ **C** $a_2a_4 < 0$ **D** $a_2 < 0$ **E** $a_2a_3 > 0$

S20. What is the second digit from the right of the number 11^{2004}?
A 0 **B** 1 **C** 2 **D** 3 **E** 4

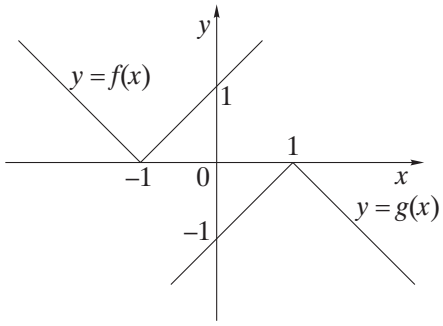
5-POINT QUESTIONS

S21. An election was held in Herbville. Every voter who voted for the Broccoli Party had already eaten broccoli. Of the remaining voters who voted for other parties 90% had never eaten broccoli. What percent did the Broccoli Party get in the election if precisely 46% of all voters in the election had eaten broccoli?
A 40% **B** 41% **C** 43% **D** 45% **E** 46%

S22. A parallelogram is divided into 4 triangles as shown in the figure. Of the following possibilities for the areas of the triangles at most one can be true. Which one is it?
A 4, 5, 8, 9 **B** 3, 5, 6, 7 **C** 5, 6, 7, 12
D 10, 11, 12, 19 **E** 5, 6, 8, 10



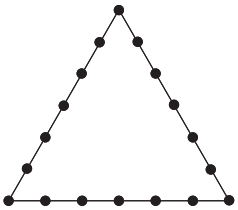
S23. The figure shows graphs of functions f and g defined on real numbers. Each graph consists of two perpendicular halflines. Which equality is satisfied for every real number x?
A $f(x) = -g(x) + 2$
B $f(x) = -g(x) - 2$
C $f(x) = -g(x + 2)$
D $f(x + 2) = -g(x)$
E $f(x + 1) = -g(x - 1)$



S24. An equilateral triangle ABC with sides of length 4 is given. The radius of the circular arc, with center at A, which divides the triangle into two parts of equal area is:
A $\sqrt{\frac{12\sqrt{3}}{\pi}}$ **B** $\sqrt{\frac{24\sqrt{3}}{\pi}}$ **C** $\sqrt{\frac{30\sqrt{3}}{\pi}}$ **D** $\frac{6\sqrt{3}}{\pi}$ **E** $\sqrt{\frac{48\sqrt{3}}{\pi}}$

S25. A game starts with a sequence of two hundred zeroes. In the first round we add 1 to every number. In the second round we add 1 to the second number and to every second number after it. In the third round we add 1 to the third number and to every third number after it, and so on. What number is in the 120th position after two hundred rounds?
A 16 **B** 12 **C** 20 **D** 24 **E** 32

S26. How many triangles can be drawn with vertices in the 18 points shown in the figure?



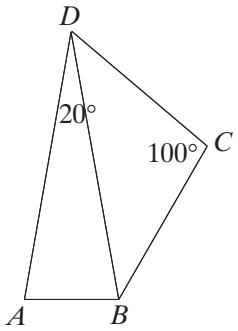
A 816 **B** 711 **C** 777 **D** 717 **E** 811

S27. If the sum of all the numbers that can be formed by permutation of the three different digits $0 < a < b < c$ is 1554, what is the value of c?
A 3 **B** 4 **C** 5 **D** 6 **E** 7

S28. The number $m = 999\ldots9$ consists of 999 nines. What is the sum of the digits of m^2 ?
A 8982 **B** 8991 **C** 9000 **D** 9009 **E** 9018

S29. $\sin^8 75^\circ - \cos^8 75^\circ$ is equal to:
A $\frac{\sqrt{3}}{2}$ **B** $\sqrt{3}$ **C** $\frac{7\sqrt{3}}{16}$ **D** 1 **E** 0

S30. Let $ABCD$ be a convex quadrilateral with an area of 1 where AB and BD are the bases of two isosceles triangles ADB and BCD respectively (as shown).



The product $AC \cdot BD$ is equal to:
A $\frac{\sqrt{3}}{3}$ **B** $\frac{2\sqrt{3}}{3}$ **C** $\sqrt{3}$ **D** $\frac{4\sqrt{3}}{3}$ **E** other answer