

1.

1:5, a 1:4, 1:k 1:2. ka 95

) ?
) ?

.)
 20, 25 50 , 5

100 ,
 20 , 25
 50 , $20 + 25 + 50 = 95$,
 95 .

2.

80%

?

$$\begin{aligned}
 & y \cdot 100x + y \cdot 0,2y \cdot x \cdot 80\% \cdot 20\% = \frac{1}{5} \cdot 5(20y + x) = 100x + y, \quad 99y = 95x. \\
 & , 95 | 99y \quad 99 | 95x, \quad \text{NZD}(95,99) = 1 \quad 0 < y < 100 \quad (?), \\
 & \quad y = 95 \quad x = 99.
 \end{aligned}$$

3. 756 ,

$$\begin{aligned}
 & \frac{1}{4} \cdot 150 \cdot ? \\
 & x \geq 4 \quad \left(\frac{756}{x} \right) \\
 & 756 = 2^2 \cdot 3^3 \cdot 7 \\
 & 4, \quad x \quad 3^3 \cdot 7. \\
 & \frac{756}{x} + 3 \cdot \frac{1}{4} \cdot \frac{756}{x} = \frac{7}{4} \cdot \frac{756}{x} = \frac{3^3 \cdot 7^2}{x} \\
 & \frac{3^3 \cdot 7^2}{x} \geq 150, \quad x \leq \frac{3^3 \cdot 7^2}{150} < 9. \\
 & \quad 4 \quad 9 \quad x = 7. \\
 & \quad 7 \quad , \quad \frac{3^3 \cdot 7^2}{7} = 189
 \end{aligned}$$

4. ,

$\frac{1}{3}$, $\frac{2}{5}$ -
 , 1 45 , -
 ?
 $\frac{2}{6}$, $\frac{2}{5}$
 $5x$, $2x$, $6x$,
 $6x + 5x - 2x = 9x$.
 $3x$, $9x - 2,5x = 6,5x$ -
 $1 \text{ h } 45 \text{ min} = 1\frac{3}{4} \text{ h}$
 $6,5x - 3x = 1\frac{3}{4}$, $x = \frac{1}{2} \text{ h}$,
 $2 \cdot \frac{1}{2} = 1 \text{ h}$.

5.

345 , 3 , -
 4 , -
 ?
 $345 = 3 \cdot 5 \cdot 23$,
 $3 \cdot 23 = 69$
 $69 - 3 = 66$.
 $3 \cdot 5 = 15$, $5 \cdot 23 = 115$
 $\text{NZD}(12,112) = 4$.

6.

$4, 3, 2$: „ 2 ,
 3 ,
 $4, 5$ „
 $\text{NZS}(2,3,4) = 12$.

$x + 12k = 2x$, \dots $x = 12k$.
 $24k < 100$, $12k$, $k \leq 4$.
 $24, 48, 72, 96$
 $12, 24, 36, 48$
 $(12, 24)$

7.

2
 2
 2 ?
 n m
 N ,
 M . $N = M + 2$. k
 $(M + 5m + k) - (N + 5n) = 2$,
 $k = 5(n - m) + 4$, $0 \leq k \leq 5$, $n = m$

$k = 4$.

$$(N + 3n) - (M + 3m + 2) = (N - M) + 3(n - m) - 2 = 0.$$

k

$$(M + 5m) - (N + 5n + k) = 2,$$

$$k + 4 = 5(m - n). \quad , \quad 0 \leq k \leq 5 \quad m = n + 1$$

$k = 1$.

$$(M + 3m) - (N + 3n) = 3(m - n) - (N - M) = 3 - 2 = 1.$$

8.

40%

$\frac{?}{abc}$

(

$\overline{acb}, \overline{bca}, \overline{bac}, \overline{cab}, \overline{cba}$)

$$60\% \overline{abc} = \frac{3}{5} \overline{abc}.$$

180

$$2(1+2+3+4+5+6+7+8+9+10+11+12) = 156 \quad 180 - 156 = 2 \cdot 12$$

13

2, 3, 4, 5, 6, 7,

8, 9, 10, 11, 12 13

9.

?

a, b, c

$$\overline{abc} = 100a + 10b + c.$$

$$\overline{abc} + \frac{1}{5} \overline{abc} = 120a + 12b + \frac{6}{5}c,$$

$$c = 0,$$

$$c = 0 \quad c = 5.$$

$$\overline{ab0} = 100a + 10b.$$

20%

$$c = 0$$

20%

$$\overline{ab0} \quad \overline{ab0} \geq \overline{a0b}$$

$$\overline{b0a} = 100b + a \quad \overline{ba0} = 100b + 10a.$$

$$119a = 88b$$

11,

11.

$$5a = 4b.$$

$a \quad b$

$$a = 4, b = 5,$$

450,

540.

$$c = 5,$$

$$\overline{ab5}.$$

$$\overline{a5b}, \overline{ba5}, \overline{b5a}, \overline{5ab}, \overline{5ba},$$

$$20a = 11(4 - b), \quad 1 = 11(10a - 8b), \quad 119a = 11(8b + 4),$$

$$494 = 11(10a + b), \quad 119a + 2b = 494.$$

11,

11,

$$a = 4, b = 9,$$

495,

594.

954

10.

A B.

12 km

4 h 40 min B,

$7\frac{5}{7} h$ A.

A B.

C,

$x h.$

$$x : 4\frac{2}{3} = 7\frac{5}{7} : x,$$

$x = 6 h.$

6 h.

BC = y km

$$AC = y + 12.$$

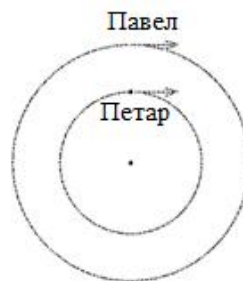
$$(y + 12) : y = 6 : 4\frac{2}{3},$$

$y = 42 km.$

$$AB = 2y + 12 = 96 km.$$

11.

28
20



$$NZS(20, 28) = 140$$

$$140 : 20 = 7 \quad , \quad 140 : 28 = 5 \quad , \quad 2$$

$$140 : 4 = 35 \quad .$$

12.

$4 \text{ km} / \text{h}$,
 $3 \text{ km} / \text{h}$.
 $6 \text{ km} / \text{h}$,
 $2 \text{ km} / \text{h}$.

8 h , 10

15 , 10 ,
 20 ,
 30 ,
 5 ,
 10 .

$$S, \quad 3S, \quad \frac{2S}{4} + \frac{S}{3} = \frac{10S}{12} = \frac{5S}{6} .$$

$$\frac{\frac{3S}{5S}}{\frac{5S}{6}} = 3,6 \text{ km} / \text{h} .$$

13.

A , B ,
 $\frac{2}{3}$,
 B , 2 h ,
 50 km ,
 A , $B ?$

$\frac{2}{3}$
 $\frac{3}{2}$
 $\frac{1}{2}$
 $2 h$ $\frac{1}{2}$ -
 $2 \cdot 2 = 4 h$
 $1 + 4 = 5 h.$
 $1 h$ $50 km$
 $6 h,$ $7 h.$ $1 h$
 $50 km$ $\frac{2}{3}$
 $\frac{3}{2}$
 $1 h$ $\frac{1}{2}$
 $50 km.$,
 $50 km$ $1 \cdot 2 = 2 h.$,
 $50 : 2 = 25 km / h$ $5 h,$ A
 B $5 \cdot 25 = 125 km.$

14. $12 km$ P $Q.$
 Q P $B,$ $5 min$ Q P
 $A.$ $1 km$ Q A $B,$ -
 P $Q.$ A B
 $2 km$ $P.$ B $P,$
 A $Q.$.
 $.$ A $1 km$ $5 min$ -
 $B.$ A $11 km$
 $55 min$ $B,$ A P $55 min$
 $B.$ A $55 + 35 = 90 min = 1,5 h.$
 A $1,5 h$ $12 km$, A
 $2 km$ P B 6 , .
 $15 min.$ A P $55 min$ $B,$
 B $2 km$ P $55 - 15 = 40 min.$

B 2 km 40 min , B -
 12 km Q P 4 h , A Q
 P $5 + 55 = 60\text{ min}$ B , A Q
 P 3 h .
 ,
 A ,
 $\frac{1}{2}(\frac{12}{1,5} - \frac{12}{3}) = 2\text{ km/h}$.

15. 6 A B
 B 16 B
 A , 16 .
 ,
 A .
 $x\text{ km/h}$ $2x\text{ km/h}$, $y\text{ km/h}$
 A B $10(x+y)\text{ km}$,
 $8(x+y)\text{ km}$, 8 $10(x+y)\text{ km}$
 $2x+y\text{ km/h}$ $2(x+y)\text{ km/h}$
 $2x-y\text{ km/h}$. $\frac{10(x+y)}{2x+y} + \frac{2(x+y)}{2x-y} = 8$, $x = 2y$.
 ,
 A $8(x+y) = 24y\text{ km}$,
 $2x-y\text{ km/h} = 3y\text{ km/h}$. , A
 $24:3 = 8\text{ h}$ A $14+8 = 22\text{ h}$.

16. a 10% $b = \overbrace{77\dots770}^m$,
 a 20% b . m .
 $a = \overbrace{855\dots547}^{m-2}$. a

20%

$$8 + 4 + 7 + 5(m - 2) = \frac{8}{10} \cdot 7m.$$

$$m = 15.$$

17.

27.

22. $\overline{abc} \cdot ab + c = 27$

$a + bc = 22.$

$$ab + c - a - bc = 5,$$

$$a(b - 1) - c(b - 1) = 5,$$

$$(a - c)(b - 1) = 5.$$

a, b, c

$$a - c = 1, b - 1 = 5 \quad a - c = 5, b - 1 = 1.$$

$$a - c = 1, b - 1 = 5 \quad b = 6, \quad 6a + c = 27 \quad a - c = 1,$$

$$a = 4, c = 3. \quad 463.$$

$$a - c = 5, b - 1 = 1 \quad b = 2, \quad 2a + c = 27, \quad a - c = 5,$$

$$a = c = 9. \quad 463.$$

$$a - c = 0 \neq 5, \dots$$

18.

207

15,

12.

NZS(12,15) = 60.

12c $60a + 15b + 12c = 207.$ $a \leq 3,$

$$(a, b, c) = (3, 1, 1), (2, 5, 1), (2, 1, 6), (1, 1, 11), (1, 5, 6), (1, 9, 1).$$

75 72 , 60 .

19.

36% 80 , 35% .
5%
25 , ,
x y ,
 $\frac{9}{25}x + \frac{7}{20}y = 80$, 25 x 20 y .
 $x = 25m$ $y = 20n$, $9m + 7n = 80$.
 $m = n = 5$. , 125

100 .
100a , a .
1,05a 125 · 1,05a .
 $100a + 100 \cdot 25 = 125 \cdot 1,05a$, $a = \frac{100 \cdot 25}{125 \cdot 1,05 - 100} = 80$.
80 ,
 $1,05 \cdot 80 = 84$,
 $\frac{125 \cdot 84 + 100 \cdot 80}{125 + 100} = \frac{740}{9} = 82 \frac{2}{9} \approx 82,22(2)$.

20.

4 , 9
10 ?
n $a_1 \geq a_2 \geq \dots \geq a_n$
 $S = a_1 + a_2 + \dots + a_n$.
 $a_1 = \frac{S-a_1}{4}$, $a_3 = \frac{S-a_3}{9}$ $a_n = \frac{S-a_n}{10}$,
 $a_1 = \frac{S}{5}$, $a_3 = \frac{S}{10}$ $a_n = \frac{S}{11}$. ,

$$\begin{aligned}
S &\leq a_1 + a_1 + a_3 + \dots + a_3 + a_n \\
&= 2a_1 + (n-3)a_3 + a_n \\
&= \frac{2S}{5} + (n-3)\frac{S}{10} + \frac{S}{11} \\
&< \frac{2S}{5} + (n-2)\frac{S}{10} = (n+2)\frac{S}{10},
\end{aligned}$$

$$\begin{aligned}
S &\geq a_1 + a_3 + a_3 + a_n + \dots + a_n + a_n \\
&= a_1 + 2a_3 + (n-3)a_n \\
&= \frac{2S}{5} + (n-3)\frac{S}{11}.
\end{aligned}$$

$$n > 8 \quad n \leq 9,6. \quad , n = 9.$$

21.

$$65 \quad 100 \quad . \quad 66 \quad ,$$

$$71 \quad , \quad 56 \quad .$$

$$5 \quad .$$

$$75 \quad , \quad ,$$

$$59 \quad .$$

$$15 \quad 30? \quad ,$$

$$. \quad x \quad , \quad y$$

$$71x + 56y ,$$

$$66(x+y) , \quad 71x + 56y = 66(x+y) ,$$

$$\dots x = 2y .$$

$$z \quad ,$$

$$66 + 5 = 71 .$$

$$75(x+z) + 59(y-z) = 71(x+y) ,$$

$$\dots y = 4z . \quad x + y = 3y = 12z \quad -$$

$$. \quad , 15 < 12z < 30 , \quad 1\frac{1}{4} < z < 2\frac{1}{2} , \quad z \in \mathbb{N}$$

$$z = 2 . \quad , \quad 12 \cdot 2 = 24 \quad -$$

22.

$$\frac{4}{3} \cdot 60 + 1 = 81$$

$$\frac{4}{3} \cdot 81 + 1 = 109, \dots$$

23.

$$1 - \left(\frac{2}{3} + \frac{3}{4} \cdot \frac{1}{3}\right) = \frac{1}{12}$$

$$1 - \left(\frac{3}{4} + \frac{2}{3} \cdot \frac{1}{4}\right) = \frac{1}{12}$$

$$12 \cdot 10 = 120$$

24.

$$42 \cdot \frac{5}{9} = 24$$

$$24 \cdot 9 = 216$$

$$216 \cdot 64\% = 138.24$$

$$138.24 = 9k$$

$$k = 15.36$$

$$25m \quad , \quad 4k + 9m \quad . \quad , \quad 5k + 16m \quad , \quad 4k + 9m = 42 \quad -$$

$$k = 6 \quad m = 2. \quad , \quad 9 \cdot 6 + 25 \cdot 2 = 104 \quad .$$

25. 20% -

$$. \quad , \quad 25%$$

$$4 \quad , \quad ?$$

$$. \quad 2 \quad x. \quad \frac{1}{5}x \quad ,$$

$$\frac{1}{4} \cdot \frac{4}{5}x = \frac{1}{5}x \quad 2 \quad , \quad .$$

$$4 \quad , \quad \frac{1}{5}x \quad 6 \quad .$$

$$x = \frac{1}{5}x + \frac{1}{5}x + 2 + \frac{1}{5}x + 6, \quad x = 20. \quad ,$$

$$20 + 2 = 22 \quad .$$

26. 1001 .

$$10% \quad , \quad 8%$$

?

$$. \quad 2x \quad , \quad 25x, \quad 20x \quad ,$$

$$y \quad , \quad y < 2x$$

$$25x + 20x - 2x = 43x, \quad 43x + y, \quad .$$

$$43x + y = 1001. \quad y < 2x \quad -$$

$$43x < 1001 < 45x, \quad 22\frac{11}{45} < x < 23\frac{12}{43}, \quad .$$

$$x = 23. \quad , 46 \quad .$$

27. 600, \quad 650

$$20 \quad , \quad 20%$$

$$25%$$

?

$$1,25(4x + 20) = 5x + 25$$

$$29x + 45, \quad 600 < 29x + 45 < 650, \quad \dots \quad 19\frac{4}{29} < x < 20\frac{25}{29}$$

$$x \in \mathbb{N} \quad x = 20.$$

$$29 \cdot 20 + 45 = 625$$

28.

$$230x + 110y + 40z = 760,$$

$$27x + 12y + 5z = 88,$$

$$x + y + z \leq 8.$$

$$x + y = 4.$$

$$27x + 12(4 - x) + 5z = 88, \quad 3x + z = 8, \quad x \leq 2.$$

$$x = 0 \quad z = 8 \quad y = 4,$$

$$x + y + z = 12 > 8. \quad x = 1 \quad z = 5 \quad y = 3,$$

$$x + y + z = 9 > 8. \quad x = 2$$

$$z = 2 \quad y = 2, \quad x + y + z = 6 \leq 8,$$

29.

$$285, \quad \frac{1}{4}$$

$$\frac{1}{5},$$

$$\frac{1}{6}, \quad \frac{1}{7}$$

?

$$12k + 35n = 285$$

$$k = 15 \quad n = 3$$

$$(1 - \frac{1}{4} - \frac{1}{6}) \cdot 180 = \frac{7}{12} \cdot 180 = 105$$

$$(1 - \frac{1}{5} - \frac{1}{7}) \cdot 105 = \frac{23}{35} \cdot 105 = 69$$

30.

2014

$z < b < c$, $c = z + x, x \geq 2$. $x = 2$,
 $z, z+1, z+2$, $\dots, 3z+3 = 2014$,
 $2014 - 3z = 3$, $x \geq 3$. $x \geq 6$,
 $z+3$, $x \leq 5$.
 $x = 5$, $z, b, z+5$,
 $z+2 < b < z+3$.
 $x = 4$, $b = z+k$ $k \in \{1, 2, 3\}$, $3z+k = 2010$
 $k = 3$, $z, z+3, z+4$

$$z+1, z+3, z+3, \quad , \quad ,$$

$$\begin{aligned} x=3, \quad b=z+k \quad k \in \{1,2\}, \quad 3z+k=2011 \\ k=1. \quad , \quad z, z+1, z+3 \\ , \quad , \quad 670, 671 \quad 673 \quad , \\ , \quad , \quad 670, 671, 673 \quad , \end{aligned}$$