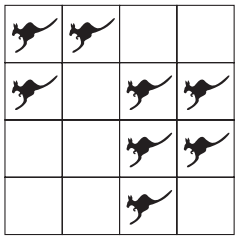
JUNIOR (grades 9 and 10)

3-POINT QUESTIONS

J1. Helga lives in her home with father, mother, brother and also one dog, two cats, two parrots and four goldfishes. How many legs do they have altogether?
A 22 **B** 24 **C** 28 **D** 32 **E** 40

J2. Sally had the fiftieth best result, and at the same time the fiftieth poorest result, at the latest Kangaroo contest in her school. How many pupils took part in the competition?
A 50 **B** 75 **C** 99 **D** 100 **E** 101

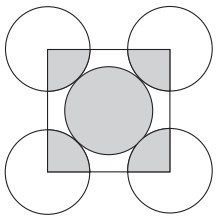
J3. In the diagram every of the eight kangaroos can jump to ane empty square. What is the least number of kangaroos that must jump so that each row and each column have exactly two kangaroos?
A 2 **B** 4 **C** 5 **D** 3 **E** 1



J4. 18 pupils are crossing a road in pairs. The pairs are labelled from 1 to 9. A pair with an even label consists of a boy and a girl, and a pair with an odd label consists of two boys. How many boys are crossing the road?
A 10 **B** 12 **C** 14 **D** 11 **E** 18

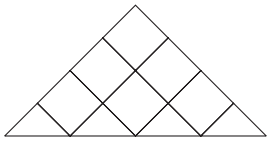
J5. Johnny inflates 8 balloons every three minutes. How many balloons will be inflated after two hours, if every tenth balloon pops immediately after having been inflated?
A 160 **B** 216 **C** 240 **D** 288 **E** 320

J6. In the diagram, the five circles have the same radius and touch as shown. The square joins the centres of the four outer circles. The ratio of the area of the shaded part of all five circles to the area of the unshaded parts of the circles is:
A 2:3 **B** 1:3 **C** 5:4 **D** 1:4 **E** 2:5

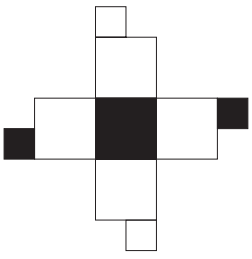


J7. Two types of bricks were produced: one of size 10 cm × 12 cm × 14 cm and another of 12 cm × 14 cm × 16 cm. In percentage, how much is the volume of the bigger brick than that of the first brick?
A 20% **B** 30% **C** 40% **D** 50% **E** 60%

J8. There are seven squares in the picture. How many more triangles than squares are there in the picture?
A 4 **B** 3 **C** 2 **D** 1 **E** 0



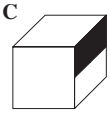
J9. Which of the following cubes has been folded out of the plan on the right?



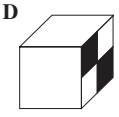
A

B

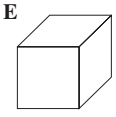
C



D



E



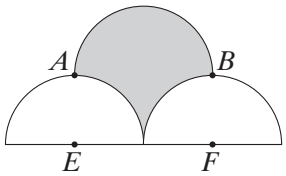
J10. A mother kangaroo and her baby Jumpy are jumping around the stadium with a perimeter of 330 m. Both of them make 1 jump every second. The mother's jumps are 5 m long, while Jumpy's jumps are 2 m long. They both start at the same point and move in the same direction. After 25 seconds Jumpy get tired and stops while his mother continues to jump. How long is it until she is next to Jumpy again?
A 15 s **B** 24 s **C** 51 s **D** 66 s **E** 76 s

4-POINT QUESTIONS

J11. What is $1+2-3-4+5+6-7-8+\cdots+2001+2002-2003-2004+2005$?
A 0 **B** 1 **C** 2005 **D** 2004 **E** -4

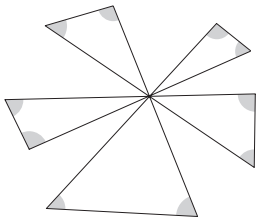
J12. For a positive integer n, by its length we mean the number of factors in the representation of n as a product of prime numbers. For example, the length of the number $90 = 2\cdot 3\cdot 3\cdot 5$ is equal to 4. How many odd numbers less than 100 have length 3?
A 7 **B** 5 **C** 3 **D** 2 **E** Another answer

J13. We are given three semi-circles as shown. $ABEF$ is a rectangle and the radius of each of the semi-circles is 2 cm. E and F are the centres of the bottom semi-circles. The area of the shaded region (in cm^2) is:
A 2π **B** 7 **C** $2\pi+1$ **D** 8 **E** $2\pi+2$



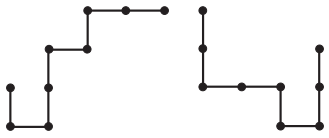
J14. Two bottles of equal volume contain both juice and water. The ratios of the volume of juice to water are, respectively, 2:1 and 4:1. We put all the contents of the two bottles into one big bottle. Then the ratio of juice to water in this bottle will be:
A 11:4 **B** 8:1 **C** 6:4 **D** 5:1 **E** 3:1

J15. What is the sum of the 10 angles marked in the picture?
A 720° **B** 600° **C** 450° **D** 360° **E** 300°



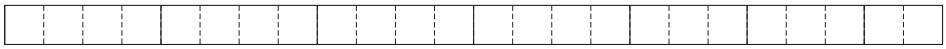
J16. The average of 16 different positive integers is 16. What is the largest possible value that one of these integers could have?
A 16 **B** 24 **C** 32 **D** 136 **E** 256

J17. Each of these two pieces of wire is made of 8 segments of length 1. One of the pieces is placed one above the other so that they coincide partially. What is the largest possible length of their common part?
A 2 **B** 3 **C** 4 **D** 5 **E** 6



J18. In a bag we have 17 balls numbered from 1 to 17. If we select some balls at random, what is the smallest number of balls needed to guarantee that the selection contains at least one pair of balls that add to 18?
A 7 **B** 8 **C** 10 **D** 11 **E** 17

J19. A rectangle with length 24 m and width 1 m is cut into smaller rectangles, each with width 1 m. There are four pieces with length 4 m, two pieces with length 3 m and one piece

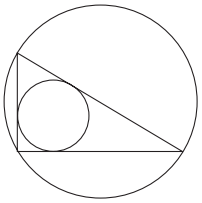


with length 2 m. These smaller rectangles are put together to form another rectangle. What is the smallest possible perimeter of the new rectangle?
A 14 m **B** 20 m **C** 22 m **D** 25 m **E** 28 m

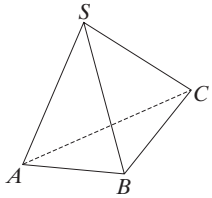
J20. A car drove with constant speed of 90 km/h. When the car clock showed 21:00, the daily mileage recorder showed 116.0, meaning that up to that moment 116.0 km had been driven. Later that evening the mileage recorder showed the same row of four ciphers as the clock. At what time did that occur?
A 21:30 **B** 21:50 **C** 22:00 **D** 22:10 **E** 22:30

5-POINT QUESTIONS

J21. Let a and b be two shorter sides of the right-angled triangle. Then the sum of the diameter of the incircle and that of the circumcircle of this triangle is equal to:
A $a+b$ **B** $2(a+b)$ **C** $0{,}5(a+b)$ **D** $\sqrt{ab}$
E $\sqrt{a^2+b^2}$

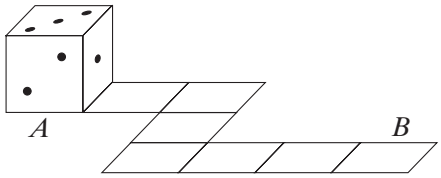


J22. In the pyramid $SABC$ all plane angles with vertex S are equal to 90°. The areas of the lateral faces SAB, SAC and SBC are 3, 4 and 6, respectively. Find the volume of $SABC$.
A 12 **B** 8 **C** 6 **D** 5 **E** 4



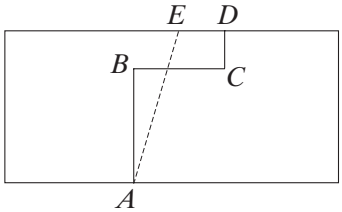
J23. Every other day Charles always speaks the truth, otherwise he lies. Today he stated exactly four of the following sentences. Which one could he not have stated today?
A I have a prime number of friends
B 288 is divisible by 12
C I have as many male friends as female
D I always speak the truth
E Three of my friends are older than me

J24. The sum the dots on opposite faces of a die always equals 7. A die rolls as shown below. At the starting point (A) the top face is 3. Which will be the face at the end point (B)?
A 2 **B** 3 **C** 4 **D** 5 **E** 6



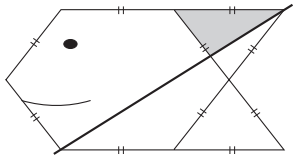
J25. How many positive integers n satisfy the inequality $2000 < \sqrt{n(n+1)} < 2005$?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

J26. Two pieces of land are separated by the borderline $ABCD$, as shown in the figure. The line segments AB, BC and CD are parallel to the sides of the rectangle and have lengths 30m, 24m and 10m, respectively. We want to straighten the borderline by replacing it with a line AE, such that the areas of the two pieces of land do not change. How far from D must be E?
A 8m **B** 10m **C** 12m **D** 14m **E** 16m

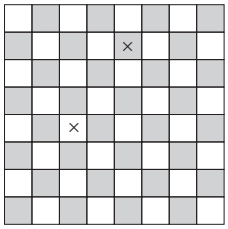


J27. How many 4-digit divisors does the number 102^2 have?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

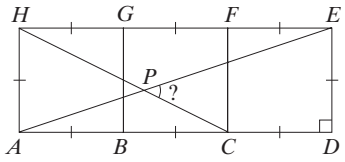
J28. Ten matches are used to make this fish-shaped figure. The piece of string is placed on the shape as shown. The area of the whole shape is 24. What is the area of the shaded triangle?
A $\sqrt{2}$ **B** $\sqrt{3}$ **C** 2 **D** $\sqrt{5}$ **E** $\sqrt{6}$



J29. How many ways are there to choose a white square and a black square from an 8×8 chess-board so that these squares lie neither in the same row nor in the same column?
A 56 **B** 5040 **C** 720 **D** 672 **E** 768



J30. Three squares are placed together as shown. The lines AE and CH intersect at point P. What is the angle $\angle CPE$?
A 30° **B** 45° **C** 60° **D** 50° **E** 40°



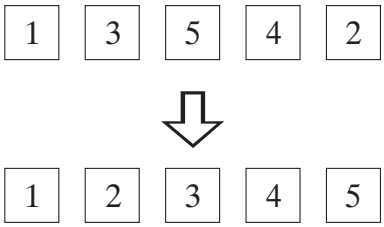
STUDENT (grades 11 and 12)

3-POINT QUESTIONS

S1. For which of the following values of x is the value of the expression $\frac{x^2}{x^3}$ the smallest?
A 1 **B** -1 **C** -2 **D** -3 **E** 100

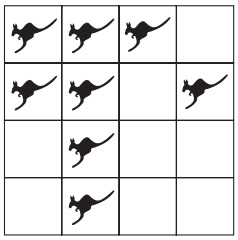
S2. How many numbers from 2 to 100 are equal to the cube of an integer?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

S3. Five cards are lying on the table in the order 1, 3, 5, 4, 2. You must get the cards in the order 1, 2, 3, 4, 5. Per move, any two cards may be interchanged. How many moves do you need at least?
A 5 **B** 4 **C** 3 **D** 2 **E** 1

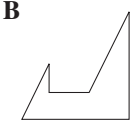
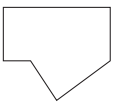
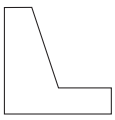
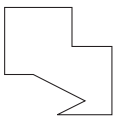


S4. If $888 \cdot 111 = 2 \cdot (2 \cdot n)^2$, and n is a positive integer, n equals:
A 8 **B** 11 **C** 22 **D** 111 **E** 444

S5. In the diagram every of the eight kangaroos can jump to any empty square. What is the least number of kangaroos that must jump so that each row and each column have exactly two kangaroos?
A 1 **B** 5 **C** 3 **D** 4 **E** 2

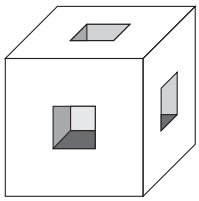


S6. A square piece of paper has been cut in three pieces. Two of them are in the picture on the right. What is the third one?

A 
B 
C 
D 
E 

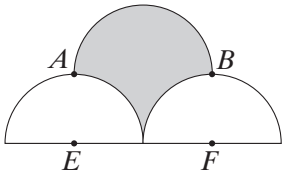
S7. The sum of four consecutive positive integers cannot be equal to:
A 2002 **B** 22 **C** 202 **D** 222 **E** 220

S8. A $3 \times 3 \times 3$ cube weighs 810 grams. If we drill three holes through it as shown, each of which is a $1 \times 1 \times 3$ rectangular parallelepiped, the weight of the remaining solid is:
A 540 g **B** 570 g **C** 600 g **D** 630 g **E** 660 g



S9. If f is a function such that $f(x+1) = 2f(x) - 2002$ holds for all integer values of x and $f(2005) = 2008$, then $f(2004)$ equals:
A 2004 **B** 2005 **C** 2008 **D** 2010 **E** 2016

S10. We are given three semi-circles as shown. $ABEF$ is a rectangle and the radius of each of the semi-circles is 2 cm. E and F are the centers of the bottom semi-circles. The area of the shaded region (in cm^2) is:
A 8 **B** 7 **C** 2π **D** $2\pi + 1$ **E** $2\pi + 2$

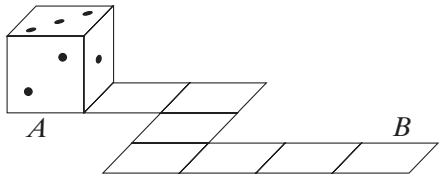


4-POINT QUESTIONS

S11. A mother kangaroo and her baby Jumpy are jumping around the stadium with a perimeter of 330 m. Both of them make 1 jump every second. The mother's jumps are 5 m long, while Jumpy's jumps are 2 m long. They both start at the same point and move in the same direction. After 25 seconds Jumpy get tired and stops while his mother continues to jump. How long is it until she is next to Jumpy again?
A 15 s **B** 24 s **C** 40 s **D** 51 s **E** 66 s

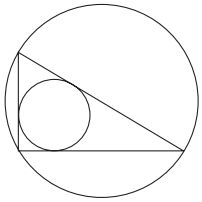
S12. Henny paints each face of several wooden cubes white or black, using both colours on each cube. How many different colourings are possible?
A 8 **B** 16 **C** 32 **D** 52 **E** 64

S13. The sum the dots on opposite faces of a die always equals 7. A die rolls as shown below. At the starting point (A) the top face is 3. Which will be the face at the end point (B)?
A 6 **B** 5 **C** 4 **D** 3 **E** 2



S14. A box contains 60 tickets: some red, some blue and some white. If all red tickets were replaced by blue tickets, then there would be twice as many blue tickets as white tickets; but if all the white tickets were replaced with blue ones, then there would be three times as many blue tickets as red tickets. The number of blue tickets in the box is:
A 10 **B** 15 **C** 20 **D** 25 **E** 30

S15. Let a and b be two shorter sides of the right-angled triangle. Then the sum of the diameter of the incircle and that of the circumcircle of this triangle is equal to:



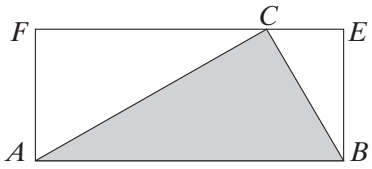
A $2(a+b)$ **B** $a+b$ **C** $0{,}5(a+b)$ **D** $\sqrt{ab}$ **E** $\sqrt{a^2+b^2}$

S16. Let M be the set of all real numbers x for which the inequality $2^{4x} < 4^{2x}$ holds. Then M is:
A $(-\infty; 1)$ **B** $(0; 1)$ **C** $(-\infty; 1) \cup (1; \infty)$ **D** $(0; \infty)$ **E** $\mathbb{R}$

S17. $1+2-3-4+5+6-7-8+\cdots+2001+2002-2003-2004+2005 =$
A 2004 **B** 2005 **C** -4 **D** 0 **E** 1

S18. Two bottles of equal volume contain both juice and water. The ratios of the volume of juice to water are, respectively, 2:1 and 4:1. We put all the contents of the two bottles into one big bottle. Then the ratio of juice to water in this bottle will be:
A 3:1 **B** 6:1 **C** 11:4 **D** 5:1 **E** 8:1

S19. The diagram shows a rectangle $ABEF$ and a triangle ABC. We know that the angle ACF equals angle CBE. If $FC = 6$ and $CE = 2$ then the area of ABC is:
A 12 **B** 16 **C** $8\sqrt{2}$ **D** $8\sqrt{3}$ **E** Another value

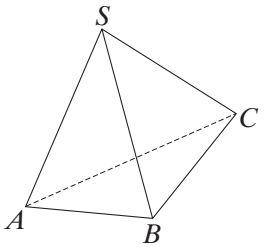


S20. Every other day Charles always says the truth, otherwise he lies. Today he stated exactly four of the following sentences. Which one couldn't he have stated today?
A I have a prime number of friends
B I have as many male friends as female
C 288 is divisible by 12
D I always say the truth
E Three of my friends are older than me

5-POINT QUESTIONS

S21. Which of the following numbers can be expressed as the product of four different integers, each of them greater than 1?
A 625 **B** 124 **C** 108 **D** 2187 **E** 2025

S22. In the pyramid $SABC$ all plane angles with vertex S are equal to 90°. The areas of the lateral faces SAB, SAC and SBC are 3, 4 and 6, respectively. Find the volume of $SABC$.
A 4 **B** 5 **C** 6 **D** 8 **E** 12



S23. If the sum of the digits of m is 30, then the sum of the digits of $m+3$ cannot be:
A 6 **B** 15 **C** 21 **D** 24 **E** 33

S24. In a bag we have 17 balls numbered by $5 + k \cdot 125$, $k = 0, \ldots, 16$, i.e. by 5, 130, 255, 380, 505, ... 1755, 1880, 2005. If we select several balls at random, what is the smallest number of balls needed to guarantee that the selection contains at least one pair of balls that add up to 2010?
A 7 **B** 8 **C** 10 **D** 11 **E** 17

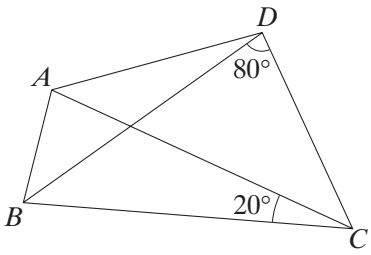
S25. If $\sqrt{2005}+\sqrt{1995}=a$, which of the following expressions has the value $\sqrt{2005}-\sqrt{1995}$?
A $10-a$ **B** $\frac{10}{a}$ **C** $\frac{a}{10}$ **D** $\frac{1}{a}$ **E** $10+a$

S26. The positive integer m has exactly two divisors. The positive integer n has exactly five divisors. How many divisors does the number $m \cdot n$ have? (The unity is a divisor. The integer itself is a divisor.)
A 5 **B** 6 **C** 7 **D** 10
E It is not possible to determine without additional information.

S27. A positive integer has k odd divisors and n even divisors. Which of the following can be the value of the quotient $\frac{n}{k}$? (The unity is a divisor. The integer itself is a divisor.)
A $\frac{1}{3}$ **B** $\frac{3}{5}$ **C** $\frac{2}{3}$ **D** 2 **E** 4

S28. Start with a number, double it and then subtract 1. After applying this procedure 98 more times (starting each time from the previous result) you get $2^{100}+1$. Which was the number you started with?
A 1 **B** 2 **C** 4 **D** 6 **E** None of these

S29. In the quadrilateral $ABCD$ the diagonal BD is the bisector of $\angle ABC$ and $AC=BC$.



Given $\angle BDC=80^\circ$ and $\angle ACB=20^\circ$, $\angle BAD$ is equal to:
A 90° **B** 100° **C** 110° **D** 120° **E** 135°

S30. Henry must travel from A to B and he plans to go at a certain speed. He would like to arrive earlier than planned and notes that travelling at a speed 5 km/h faster than planned he will arrive 5 hours earlier and travelling at a speed 10 km/h faster than planned he will arrive 8 hours earlier. What is his planned speed?
A 10 km/h **B** 15 km/h **C** 20 km/h **D** 25 km/h **E** Impossible to determine