Algebra

A1	Determine all polynomials P with real coefficients satisfying the following condition: whenever x and y are real numbers such that $P(x)$ and $P(y)$ are both rational, so is $P(x+y)$.
A2	Fix an integer $n \geq 2$ and let $a_1, \ldots, a_n$ be integers, where $a_1 = 1$. Let $f(x) = \sum_{m=1}^{n} a_m m^x.$ Suppose that $f(x) = 0$ for some K consecutive positive integer values of x. In terms of n, determine the maximum possible value of K.

Combinatorics

C1	Determine all integers $n \geq 3$ for which there exists a conguration of n points in the plane, no three collinear, that can be labelled 1 through n in two different ways, so that the following condition be satisfied: For every triple $(i,j,k), 1 \leq i < j < k \leq n$, the triangle ijk in one labelling has the same orientation as the triangle labelled ijk in the other, except for $(i,j,k) = (1,2,3)$.
C2	For positive integers $m, n \geq 2$, let $S_{m,n} = \{(i,j) : i \in \{1,2,\ldots,m\}, j \in \{1,2,\ldots,n\}\}$ be a grid of mn lattice points on the coordinate plane. Determine all pairs (m,n) for which there exists a simple polygon P with vertices in $S_{m,n}$ such that all points in $S_{m,n}$ are on the boundary of P, all interior angles of P are either $90°$ or $270°$ and all side lengths of P are 1 or 3.

Geometry

G1	Let ABC be a triangle with incentre I and circumcircle ω. The incircle of the triangle ABC touches the sides BC, CA and AB at D, E and F, respectively. The circumcircle of triangle ADI crosses ω again at P, and the lines PE and PF cross ω again at X and Y, respectively. Prove that the lines AI, BX and CY are concurrent.
G2	Let $ABCD$ be a cyclic quadrilateral. Let DA and BC intersect at E and let AB and CD intersect at F. Assume that A, E, F all lie on the same side of BD. Let P be on segment DA such that $\angle CPD = \angle CBP$, and let Q be on segment CD such that $\angle DQA = \angle QBA$. Let AC and PQ meet at X. Prove that, if $EX = EP$, then EF is perpendicular to AC.
G3	A point P is chosen inside a triangle ABC with circumcircle Ω. Let Γ be the circle passing through the circumcenters of the triangles APB, BPC, and CPA. Let Ω and Γ intersect at points X and Y. Let Q be the reflection of P in the line XY. Prove that $\angle BAP = \angle CAQ$.

Number Theory

N1	Let n be a positive integer. Let S be a set of ordered pairs (x,y) such that $1 \leq x \leq n$ and $0 \leq y \leq n$ in each pair, and there are no pairs (a,b) and (c,d) of different elements in S such that $a^2 + b^2$ divides both $ac + bd$ and $ad - bc$. In terms of n, determine the size of the largest possible set S.
N2	For every non-negative integer k let $S(k)$ denote the sum of decimal digits of k. Let $P(x)$ and $Q(x)$ be polynomials with non-negative integer coecients such that $S(P(n)) = S(Q(n))$ for all non-negative integers n. Prove that there exists an integer t such that $P(x) - 10^t Q(x)$ is a constant polynomial.