

[4] 2008 -
2005 , 2006 -
[2]
[1] -

1.

1. M

$$\begin{aligned} n \in \mathbb{N} \quad M &= 2n+1, \\ M &= 2n+1 = n^2 + 2n+1 - n^2 = (n+1)^2 - n^2, \\ M &= (n+1)^2 - n^2 \quad M = (k+1)^2 - k^2 \\ (n+1)^2 - n^2 &= (k+1)^2 - k^2, \\ 2n+1 &= 2k+1, \quad n = k, \end{aligned}$$

1.

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1,

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$$3 = 2^2 - 1^2,$$

$$6k + 1 = 2 \cdot 3k + 1 = (3k + 1)^2 - (3k)^2,$$

$$6k - 1 = 2 \cdot (3k - 1) + 1 = (3k)^2 - (3k - 1)^2,$$

$$3 \qquad 6k \pm 1,$$

$k \in \mathbb{N}$.

1

M

$n + k \quad n$

$$M, \quad n \geq 1, k \geq 2,$$

$$M = (n + k)^2 - n^2 = (n + k - n)(n + k + n) = k(2n + k),$$

M

M

$$p, \dots \quad M = p^2 \quad n + k \quad n, k \geq 1$$

M ,

$$p^2 = k(2n + k),$$

$$k = 1 \quad 2n + k = p^2,$$

$$n = \frac{p^2 - 1}{2}, k = 1,$$

M

$$n = \frac{p^2 - 1}{2}, n + 1 = \frac{p^2 + 1}{2}.$$

2.

M

p

M ,

p

$$\begin{aligned}
 & k > p \qquad M = pk \quad (- \\
 ?). \quad & a, b \in \mathbb{N} \qquad M = a^2 - b^2. \quad , \\
 & pk = (a - b)(a + b)
 \end{aligned}$$

$$a - b < a + b,$$

$$\begin{cases} a - b = p, \\ a + b = k. \end{cases}$$

$$a = \frac{k+p}{2}, b = \frac{k-p}{2} \in \mathbb{N},$$

1.

165

165

$$a^2 - b^2 = 165. \quad , \quad 165 = 3 \cdot 5 \cdot 11,$$

$$(a - b)(a + b) = 3 \cdot 5 \cdot 11,$$

$$\begin{cases} a - b = 1, \\ a + b = 165, \end{cases} \quad \begin{cases} a - b = 3, \\ a + b = 55, \end{cases} \quad \begin{cases} a - b = 5, \\ a + b = 33, \end{cases} \quad \begin{cases} a - b = 11, \\ a + b = 15, \end{cases}$$

$$a = 83, b = 82; \quad a = 29, b = 26; \quad a = 19, b = 14 \quad a = 13, b = 2,$$

165

$$165 = 83^2 - 82^2,$$

$$165 = 29^2 - 26^2,$$

$$165 = 19^2 - 14^2,$$

$$165 = 13^2 - 2^2.$$

3.

M

$$4 \mid M.$$

• $4 \mid M$, k $M = 4k$,

$$(k+1)^2 - (k-1)^2 = (k+1+k-1)(k+1-k+1) = 2k \cdot 2 = 4k = M.$$

, M $4 \nmid M$, k

$M = 4k + 2$.

$$M = x^2 - y^2 = (x-y)(x+y),$$

M , $x-y$ $x+y$.
 , $x-y$ $x+y$, M $2 \cdot 2 = 4$,

4. $M = 4k$
?

$M = 4k, k \in \mathbb{N}$ $m, n \in \mathbb{N}$

$$(2m)^2 - (2n)^2 = 4k.$$

$m^2 - n^2 = k$, 1 k ,

, 3 k ,
 $4 \mid k$,

2. M

$M = 16s$

$M = 8s + 4, s \in \mathbb{N}$.

2.) $24 = 4 \cdot 6$, 3 24

, 3 .
 $24 = 4 \cdot 6 = (6+1)^2 - (6-1)^2 = 7^2 - 5^2$.

, $24 = 5^2 - 1^2$,

24) , $24 = 8 \cdot 3 + 0$, 2 24 .

) $20 = 8 \cdot 2 + 4,$ 2 20

$$\begin{aligned} 20 &= 4 \cdot 5 = 4 \cdot (2 \cdot 2 + 1) = 4 \cdot [(2+1)^2 - 2^2] \\ &= 2^2 \cdot (3^2 - 2^2) = (2 \cdot 3)^2 - (2 \cdot 2)^2 \\ &= 6^2 - 4^2. \end{aligned}$$

2.

4. $n \in \mathbb{N}.$

$$x^2 + y^2 = n \tag{1}$$

$$n = 4j + 3 \quad t \quad (\quad) \quad p^t, \quad p$$

3.) $19 = 4 \cdot 4 + 3$ 19 4
 $19^{2k+1}, k = 0, 1, 2, \dots$

) $21 = 3 \cdot 7$ $3 = 4 \cdot 0 + 3, 7 = 4 \cdot 1 + 3,$ 4 $-$
 21 $-$

) $29 = 4 \cdot 7 + 1$ 29 4 $-$
 29 $-$
 $29 = 2^2 + 5^2.$

4.) $52 = 2^2 \cdot 13$ $13 = 4 \cdot 3 + 1,$ 4
 52

$$52 = 4 \cdot 13 = 2^2 \cdot (3^2 + 2^2) = (2 \cdot 3)^2 + (2 \cdot 2)^2 = 6^2 + 4^2.$$

) $74 = 2 \cdot 37$ $37 = 4 \cdot 9 + 1$, 4
 74 .
 , $74 = 7^2 + 5^2$.

1. 105

2. $4n + 2$

3. 1105

4. 2295

5. -

$\frac{k}{2}, k \in \mathbb{N}$.

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2. , .

3. , . . , , ,

4. , . 2008 , ,

5. , . , ,