

$4 \ 5, \quad 5 \ 6. \quad 4 \ 5$

$4, 5, 6$

$n \quad k$

$n, n+1, n+2, \dots, n+k$

$(\quad 4).$

n

$(\quad n).$

$(\quad),$

$“ \quad ” \quad “ \quad ”$

$N = \{1, 2, 3, \dots, n, n+1, \dots\}.$

1.

$1, 2, 3, 4, 5, 6, \dots,$ “

$: 1+2=3,$

$“ \quad ”$

$“ \quad ”$

$4+5+6=7+8,$

$“ \quad ”$

$“ \quad ”$

$“ \quad ?”$

$!$

n

$n+(n+1)+(n+2)=(n+3)+(n+4).$

$3n+3=2n+7,$

n

$: n=4.$

$: 4+5+6=7+8,$

$(\quad “ \quad ”!):$

I

$1+2=3$

$4+5+6=7+8$

$9+10+11+12=13+14+15$

$16+17+18+19+20=21+22+23+2$

“ ”
 n
 $n + (n+1) + (n+2) + \dots + (n+k) = (n+k+1) + (n+k+2) + \dots + (n+2k)$,
 $k+1 \dots$, $k \dots$
 $n = k^2$. , $k-$.1 k^2 .
 :

2.
 $3^2 + 4^2 = 5^2$
 $10^2 + 11^2 + 12^2 = 13^2 + 14^2$
 $21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$
 $36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$

 21 . 2 “
 (.
 2) . : “
 ?”

 n

 $n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 = (n+4)^2 + (n+5)^2 + (n+6)^2$.

 $n^2 - 18n - 63 = 0$, . . . $(n-21) \cdot (n+3) = 0$,
 $n = 21$ $n = -3$. n , $(n = -3)$
 , $n = 21$.
 . 2 “
 ”.

2.
 , -

 :

- 1) n $3n^2 + n + 1$?
- 2) n , $b = 2 \cdot (1 + 9 + 9^2 + \dots + 9^n)$
 ?

..., n , ..., $n, n+1$, ...

:

1.Ě.

...

2.Ě.

$$n > 1, \quad \frac{1}{4}(n^2 - 1)$$

...

. $n = 2k + 1, k \in N$. :

$$\frac{1}{4}(n^2 - 1) = \frac{1}{4}((2k + 1)^2 - 1) = \frac{1}{4}(4k^2 + 4k) = k \cdot (k + 1).$$

:

3.Ě.

$$n, \quad n(n+1), (n+1)(n+2)$$

...

4.Ě.

$$\dots : 0, 2 \quad 6.$$

. ... :

$$1 \cdot 2 = 2, \quad 2 \cdot 3 = 6, \quad 3 \cdot 4 = 12, \quad 4 \cdot 5 = 20, \quad 5 \cdot 6 = 30, \\ 6 \cdot 7 = 42, \quad 7 \cdot 8 = 56, \quad 8 \cdot 9 = 72, \quad 9 \cdot 10 = 90, \quad 10 \cdot 11 = 110.$$

$$0, 2 \quad 6, \quad ,$$

:

$$11 \cdot 12 = 132, \quad 12 \cdot 13 = 156, \quad 13 \cdot 14 = 182, \quad 14 \cdot 15 = 210$$

, , ... 0, 2 6. -

5.Ě.

$$\dots 6, \quad 18$$

2.

. n :

$$6k, \quad 6k + 1, \quad 6k + 2, \quad 6k + 3, \quad 6k + 4, \quad 6k + 5 \quad (k = 0, 1, 2, \dots).$$

$$n(n+1), \quad :$$

$$6 \cdot k(6k + 1), \quad (6k + 1)(6k + 2) = 18 \cdot (2k^2 + k) + 2,$$

$$(6k + 2)(6k + 3) = 6 \cdot (3k + 1)(2k + 1), \quad (6k + 3)(6k + 4) = 6 \cdot (2k + 1)(3k + 2),$$

$$(6k + 4)(6k + 5) = 18 \cdot (2k^2 + 3k + 1) + 2, \quad (6k + 5)(6k + 6) = 6 \cdot (6k + 1)(k + 1).$$

$$\dots 6,$$

$$18 \quad 2.$$

3.

$$1^\circ - 5^\circ$$

1.

$$n,$$

$$n^3 + 2n^2 + 3n + 5 \quad \dots$$

$n^3 + 2n^2 + 3n + 5 = 8k^3 + 8k^2 + 6k + 5 = 2 \cdot (4k^3 + 4k^2 + 3k) + 5$
 $n^3 + 2n^2 + 3n + 5 = (2k+1)^3 + 2 \cdot (2k+1)^2 + 3 \cdot (2k+1) + 5 = 2 \cdot (4k^3 + 10k^2 + 10k) + 11$

2. $B = 2 \cdot (1 + 9 + 9^2 + \dots + 9^{n-1})$
 $B = 2 \cdot \frac{9^n - 1}{9 - 1} = \frac{1}{4} \cdot ((3^n)^2 - 1) = \frac{1}{4} (m^2 - 1), \quad m = 3^n$

$B = 2 \cdot \frac{(1 + 9 + \dots + 9^{n-1})(9 - 1)}{9 - 1} = 2 \cdot \frac{9^n - 1}{8} = \frac{1}{4} \cdot ((3^n)^2 - 1) = \frac{1}{4} (m^2 - 1)$
 $n \in \mathbb{N}$

3. $n \in \mathbb{N}, \quad B = n^2 + 9n + 16$
 $(n+3)(n+4) < B < (n+4)(n+5)$
 $n^2 + 7n + 12 < n^2 + 9n + 16 < n^2 + 9n + 20$

4. $n \in \mathbb{N}, \quad 2 \cdot (6^n + 1)$
 $n \in \mathbb{N}, \quad 6^n$
 $2 \cdot (6^n + 1)$

5. $n \in \mathbb{N}, \quad 3n + 1$
 $3n + 1 = 18q + 2$
 $3n + 1 = 18q + 1$
 $3n + 1 = 18q + 1$

6. $B = n^2 + 4n + 12$

$(n+1)(n+2) = n^2 + 3n + 2 \leq B$, $(n+1)(n+2)$
 $(n+2)(n+3)$. . . ,
 n
 $n^2 + 4n + 12 \geq (n+2)(n+3) = n^2 + 5n + 6$, . . . $n \leq 6$.
, n B , 1° ,
: $n = 2; 47; 6$. $n = 2$,
24, $n = 4$, $42 = 6 \cdot 7$ $n = 6$, $72 = 8 \cdot 9$.
, {4, 6}.
7. $6^x = y^2 + y - 2$ **N**.
. $6^x = y^2 + y - 2$: $6^x + 2 = y(y+1)$.
 6^x 6, $6^x + 2$ 8.
4°, **N**.

4.

1. 11 . . . 6
5.
2. .1 .
3. $10^2 + 11^2 + 12^2 = 13^2 + 14^2$ (.2),
.
4. .2. .
5. n k - (. . k -) .2?
: k - $(k+1) + k = 2k + 1$ (. . .)
 $n^2 + (n+1)^2 + \dots + (n+k)^2 = (n+k+1)^2 + \dots + (n+k+k)^2$,
. . $n^2 - 2k^2n - k^2(2k+1) = 0$. (: $n = k(2k+1)$.)
6. n :
) $n^2 + n + 3$;) $n^3 - 4n + 3n + 5$
. . . .
7. , $n \in N$, :
) $6 \cdot (1 + 25 + 25^2 + \dots + 25^{n-1})$;) $12 \cdot (1 + 49 + 49^2 + \dots + 49^n)$;
) $\frac{1}{4}(n^4 + 2n^3 + 3n^2 + 2n)$
8. , $n \in N$, :
) $n^2 + 3n + 4$;) $n^4 + 7n^2 + 10$

-
9. \dots
 $n \in N$ $B(n)$ \dots ,
 $B(n)$:
) $n^2 + 5n + 6$;) $n^4 + 13n^2 + 42$;) $n^2 + 3n + 5$;
) $n^2 + 4n + 5$;) $n^2 + 2n + 12$;) $n^2 + 4n + 11$;
) $n^2 + 2n + 18$.
 10. N :
) $5^x = y^2 + y - 3$;) $6^x = y^2 + 5y + 4$.
 11. $n \in N$, $3 \cdot (4^n + 2)$
 \dots
 12. $n \in N$ $n! + 4$ \dots
 13. $n \in N$, $9n + 1$ \dots
 14. $n \in N$, $9n + 2$ \dots
 n \dots
 :

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