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**1** For a real number $\alpha > 0$, consider the infinite real sequence defined by $x_1 = 1$ and

$$\alpha x_n = x_1 + x_2 + \cdots + x_{n+1} \qquad \text{for } n \geq 1.$$

Determine the smallest $\alpha$ for which all terms of this sequence are positive reals.
(Proposed by Gerhard Woeginger, Austria)

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**2** In an acute $\triangle ABC$, prove that

$$\frac{1}{3}\left(\frac{\tan^2 A}{\tan B \tan C} + \frac{\tan^2 B}{\tan C \tan A} + \frac{\tan^2 C}{\tan A \tan B}\right) + 3\left(\frac{1}{\tan A + \tan B + \tan C}\right)^{\frac{2}{3}} \geq 2.$$

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**3** Consider a binary matrix $M$(all entries are 0 or 1) on $r$ rows and $c$ columns, where every row and every column contain at least one entry equal to 1. Prove that there exists an entry $M(i, j) = 1$, such that the corresponding row-sum $R(i)$ and column-sum $C(j)$ satisfy $rR(i) \geq cC(j)$.
(Proposed by Gerhard Woeginger, Austria)

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**4** Let $O$ be the circumcenter,$R$ be the circumradius, and $k$ be the circumcircle of a triangle $ABC$ .
Let $k_1$ be a circle tangent to the rays $AB$ and $AC$, and also internally tangent to $k$.
Let $k_2$ be a circle tangent to the rays $AB$ and $AC$ , and also externally tangent to $k$. Let $A_1$ and $A_2$ denote the respective centers of $k_1$ and $k_2$.
Prove that: $(OA_1 + OA_2)^2 - A_1A_2^2 = 4R^2$.

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