
1.

$$(1) \quad \begin{cases} x + y = 30 \\ xy = 209. \end{cases}$$

$$t^2 - 30t + 209 = 0.$$

19 11.

$x,$ $y,$

:

$$x_1 = 19, \quad y_1 = 11 \quad x_2 = 11, \quad y_2 = 19.$$

2.

$$\begin{cases} x + xy + y = 11 \\ x^2y + xy^2 = 30. \end{cases}$$

$$\begin{cases} (x + y) + xy = 11 \\ (x + y)xy = 30 \end{cases}$$

$$\begin{cases} x + y = u \\ xy = v \end{cases}$$

$$\begin{cases} u + v = 11 \\ uv = 30 \end{cases}$$

$$u_1 = 6, v_1 = 5 \quad u_2 = 5, v_2 = 6. \quad ,$$

$$\begin{cases} x + y = 6 \\ xy = 5 \end{cases} \quad \begin{cases} x + y = 5 \\ xy = 6 \end{cases}$$

:

$$x_1 = 5, y_1 = 1; \quad x_2 = 1, y_2 = 5; \quad x_3 = 3, y_3 = 2; \quad x_4 = 2, y_4 = 3.$$

_____ **3.**

$$(2) \quad \begin{cases} x+y=a \\ xy=b, \end{cases} \quad a, b \in R.$$

_____.

$$t^2 - at + b = 0,$$

(1),

(2):

$$x_1 = \frac{a + \sqrt{a^2 - 4b}}{2}, y_1 = \frac{a - \sqrt{a^2 - 4b}}{2}; \quad x_2 = \frac{a - \sqrt{a^2 - 4b}}{2}, y_2 = \frac{a + \sqrt{a^2 - 4b}}{2}.$$

_____ **4.**

$$\begin{cases} (x+1)^2(y+1)^2 = 27xy \\ (x^2+1)(y^2+1) = 10xy. \end{cases}$$

_____.

$$xy \neq 0.$$

xy

$$\begin{cases} (x + \frac{1}{x} + 2)(y + \frac{1}{y} + 2) = 27 \\ (x + \frac{1}{x})(y + \frac{1}{y}) = 10. \end{cases}$$

,

$$x + \frac{1}{x} = u; \quad y + \frac{1}{y} = v,$$

$$\begin{cases} (u+2)(v+2) = 27 \\ uv = 10, \end{cases}$$

$$\begin{cases} u+v = \frac{13}{2} \\ uv = 10 \end{cases}$$

(2).

$$u_1 = 4, v_1 = \frac{5}{2}; \quad u_2 = \frac{5}{2}, v_2 = 4,$$

,

:

$$x_{1/2} = 2, y_{1/2} = 2 \pm \sqrt{3}; \quad x_{3/4} = \frac{1}{2}, y_{3/4} = 2 \pm \sqrt{3}; \quad x_{5/6} = 2 \pm \sqrt{3}, y_{5/6} = 2;$$

$$x_{7/8} = 2 \pm \sqrt{3}, y_{7/8} = \frac{1}{2}.$$

5.

$$\begin{cases} x^3 - y^3 = 19(x - y) \\ x^3 + y^3 = 7(x + y). \end{cases}$$

_____.

$$(1) \quad \begin{cases} (x - y)(x^2 + xy + y^2 - 19) = 0 \\ (x + y)(x^2 - xy + y^2 - 7) = 0. \end{cases}$$

) $x - y = 0 \Leftrightarrow x = y.$

$$2x(x^2 - x^2 + x^2 - 7) = 0 \Rightarrow x_1 = 0, y_1 = 0; \quad x_{2/3} = y_{2/3} = \pm\sqrt{7}.$$

) $x + y = 0 \Leftrightarrow y = -x.$

$$2x(x^2 - x^2 + x^2 - 19) = 0$$

(_____))

$$x_{4/5} = \pm\sqrt{19}, y_{4/5} = \mp\sqrt{19}.$$

) $y \neq x \quad y \neq -x. \quad (3)$

$$\begin{cases} x^2 + xy + y^2 = 19 \\ x^2 - xy + y^2 = 7, \end{cases} \quad \cdot \cdot \quad \begin{cases} x^2 + y^2 = 13 \\ xy = 6, \end{cases}$$

$$x_{6/7} = \pm 3, y_{6/7} = \pm 2; \quad x_{8/9} = \pm 2, y_{8/9} = \pm 3.$$

6.

$$\begin{cases} x^4 + y^4 = b^4 \\ x + y = a, \quad ab \neq 0. \end{cases}$$

_____.

$$t = x - y.$$

$$x = \frac{a+t}{2}, y = \frac{a-t}{2}.$$

$$(a+t)^4 + (a-t)^4 = 16b^4,$$

$$t^4 + 6a^2t^2 = 8b^4 - a^4.$$

$$t^2 = v,$$

$$v^2 + 6a^2v + (a^4 - 8b^4) = 0$$

.

$$\begin{cases} x + y - \sqrt{xy} = 7 \\ (x + y)\sqrt{xy} = 78 \end{cases}$$

$$x + y = u, \sqrt{xy} = v$$

$$\begin{cases} u - v = 7 \\ uv = 78 \end{cases} \quad \begin{cases} u^2 - 2uv + v^2 = 49 \\ 2uv = 156. \end{cases}$$

$$\begin{cases} u^2 + v^2 = 205 \\ 2uv = 156 \end{cases} \quad \begin{cases} (u + v)^2 = 361 \\ (u - v)^2 = 49. \end{cases}$$

:

$$) \begin{cases} u + v = 19 \\ u - v = 7 \end{cases} \quad u = 13, v = 6, \quad \dots \begin{cases} x + y = 13 \\ xy = 36 \end{cases}$$

$$: x_1 = 9, y_1 = 4; \quad x_2 = 4, y_2 = 9.$$

$$) \begin{cases} u + v = -19 \\ u - v = 7 \end{cases} \quad u = -6, v = -13. \quad \sqrt{xy} = v,$$

$$) \begin{cases} u + v = 19 \\ u - v = -7 \end{cases} \quad u = 6, v = 13. \quad \begin{cases} x + y = 6 \\ xy = 169 \end{cases} \quad -$$

$$) \begin{cases} u + v = -19 \\ u - v = -7 \end{cases} \quad v < 0, \quad (\quad)$$

10.

$$\begin{cases} 7\sqrt[3]{xy} - 3\sqrt{xy} = 4 \\ x + y = 20. \end{cases}$$

$$\sqrt[6]{xy} = t,$$

$$3t^3 - 7t^2 + 4 = 0 \Leftrightarrow (t - 1)(3t^2 - 4t - 4) = 0$$

$$t_1 = 1, \quad t_2 = 2, \quad t_3 = -\frac{2}{3}.$$

(2)

:

$$x_{1/2} = 10 \pm 3\sqrt{11}, y_{1/2} = 10 \mp 3\sqrt{11};$$

$$x_3 = 16, y_3 = 4;$$

$$x_4 = 4, y_4 = 16;$$

$$x_{5/6} = 10 \pm \frac{2\sqrt{18209}}{27}, y_{5/6} = 10 \mp \frac{2\sqrt{18209}}{27}.$$

11.

$$\begin{cases} \sqrt[3]{x} + \sqrt[3]{y} = 4 \\ xy = 27. \end{cases}$$

$$x + y + 3x^{\frac{2}{3}}y^{\frac{1}{3}} + 3x^{\frac{1}{3}}y^{\frac{2}{3}} = 4^3 \Leftrightarrow x + y + 3\sqrt[3]{xy}(\sqrt[3]{x} + \sqrt[3]{y}) = 4^3,$$

$$\begin{cases} x + y = 28 \\ xy = 27 \end{cases}$$

$$\therefore x_1 = 27, y_1 = 1; \quad x_2 = 1, y_2 = 27.$$

12.

$$\begin{cases} x^2 + y^2 + z^2 = 14 \\ (x^2 + y^2)^2 + (y^2 + z^2)^2 + (x^2 + z^2)^2 = 294 \\ \frac{(x^2 + y^2)(x^2 + z^2)}{y^2 + z^2} = \frac{50}{13}. \end{cases}$$

$$\begin{cases} x^2 + y^2 = u \\ x^2 + z^2 = v \\ y^2 + z^2 = t \end{cases}$$

$$\begin{cases} u + v + t = 28 \\ u^2 + v^2 + t^2 = 294 \\ \frac{uv}{t} = \frac{50}{13} \end{cases}$$

$$\begin{cases} u + v = 28 - t \\ (u + v)^2 - 2uv + t^2 = 294 \\ uv = \frac{50}{13}t. \end{cases}$$

$$(28 - t)^2 - 2 \cdot \frac{50}{13}t + t^2 = 294$$

$$t_1 = 13 \quad t_2 = \frac{245}{13} \quad t,$$

$$u \quad v, \quad x, y, z. \quad :$$

$$x_{1/2} = \pm 1, y_{1/2} = \pm 3, z_{1/2} = \pm 2;$$

$$x_{3/4} = \pm 1, y_{3/4} = \pm 2, z_{3/4} = \pm 3.$$

13.

$$\begin{cases} x + y + z = 1 \\ x^2 + y^2 + z^2 = 1 \\ x^3 + y^3 + z^3 = 1. \end{cases}$$

:

$$\lambda_1 = x + y + z; \quad \lambda_2 = xy + xz + yz; \quad \lambda_3 = xyz,$$

$$\lambda_1 = 1, \quad \lambda_1^2 - 2\lambda_2 = 1, \quad \lambda_1^3 - 3\lambda_1\lambda_2 + 3\lambda_3 = 1 (= x^3 + y^3 + z^3),$$

$$\lambda_1 = 1, \quad \lambda_2 = 0, \quad \lambda_3 = 0,$$

$$\begin{cases} x + y + z = 1 \\ xy + xz + yz = 0 \\ xyz = 0. \end{cases}$$

$$x, \quad y \quad z=0,$$

$$x_1 = 0, \quad y_1 = 0, \quad z_1 = 1; \quad x_2 = 0, \quad y_2 = 1, \quad z_2 = 0; \quad x_3 = 1, \quad y_3 = 0, \quad z_3 = 0.$$

:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz) \Rightarrow$$

$$1 - 3xyz = 1 - xy - xz - yz \Leftrightarrow 3xyz = xy + xz + yz =$$

$$= \frac{1}{2}[(x + y + z)^2 - (x^2 + y^2 + z^2)] = \frac{1}{2}(1 - 1) = 0.$$

$$xyz = 0, \quad x, \quad y \quad z=0.$$

$$x = 0,$$

$$\begin{cases} y + z = 1 \\ y^2 + z^2 = 1 \\ y^3 + z^3 = 1. \end{cases}$$

()

$$\begin{cases} y^2 + z^2 + 2yz = 1 \\ y^2 + z^2 = 1 \end{cases}$$

$$yz = 0,$$

$$y = 0,$$

$$z = 0.$$

14.

$$\begin{cases} x + y + z = 9 \\ xy + xz + yz = 26 \\ xyz = 24. \end{cases}$$

_____:

$$t^3 - 9t^2 + 26t - 24 = 0$$

$$t_1 = 2, \quad t_2 = 3, \quad t_3 = 4,$$

(x, y, z)

:

$$(2, 3, 4), (2, 4, 3), (3, 2, 4), (3, 4, 2), (4, 2, 3), (4, 3, 2).$$

15.

$$\begin{cases} x + y + z = 6 \\ x^2 + y^2 + z^2 = 18 \\ \sqrt{x} + \sqrt{y} + \sqrt{z} = 4. \end{cases}$$

_____:

,

$$(4) \quad xy + xz + yz = 9.$$

$$(5) \quad \sqrt{xy} + \sqrt{xz} + \sqrt{yz} = 5.$$

$$(5) \quad (4),$$

$$(\sqrt{x} + \sqrt{y} + \sqrt{z})\sqrt{xyz} = 8.$$

$$\sqrt{xyz} = 2, \dots$$

$$(6) \quad xyz = 4.$$

(4), (5) (6),

$$\begin{cases} x + y + z = 6 \\ xy + xz + yz = 9 \\ xyz = 4, \end{cases}$$

14.

:

$$x_1 = 1, \quad y_1 = 1, \quad z_1 = 4; \quad x_2 = 1, \quad y_2 = 4, \quad z_2 = 1; \quad x_3 = 4, \quad y_3 = 1, \quad z_3 = 1.$$