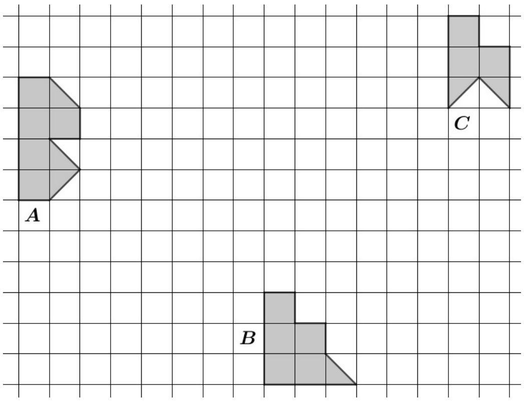
11th Iranian Geometry Olympiad

Elementary level

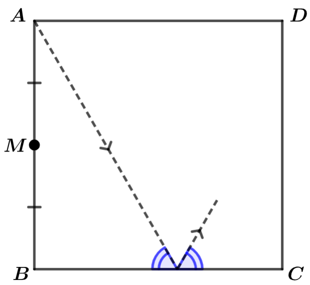
October 18, 2024

The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.com

Problem 1. Reflect each of the shapes A, B over some lines l_A, l_B respectively and rotate the shape C such that a 4×4 square is obtained. Identify the lines l_A, l_B and the center of the rotation, and also draw the transformed versions of A, B and C under these operations.



Problem 2. $ABCD$ is a square with side length 20. A light beam is radiated from A and intersects sides BC, CD, DA respectively and reaches the midpoint of side AB. What is the length of the path that the beam has taken?



Problem 3. Inside a convex quadrilateral $ABCD$ with $BC > AD$, a point T is chosen. S lies on the segment AT such that $DT = BC, \angle TSD = 90°$. Prove that if $\angle DTA + \angle TAB + \angle ABC = 180°$, then $AB + ST \geqslant CD + AS$.

Problem 4. An inscribed n-gon ($n > 3$), is divided into $n-2$ triangles by diagonals which meet only in vertices. What is the maximum possible number of congruent triangles obtained? (An inscribed n-gon is an n-gon where all its vertices lie on a circle)

Problem 5. Points Y, Z lie on the smaller arc BC of the circumcircle of an acute triangle $\triangle ABC$ (Y lies on the smaller arc BZ). Let X be a point such that the triangles $\triangle ABC, \triangle XYZ$ are similar (in this exact order) with A, X lying on the same side of YZ. Lines XY, XZ intersect sides AB, AC at points E, F respectively. Let K be the intersection of lines BY, CZ. Prove that one of the intersections of the circumcircles of triangles $\triangle AEF, \triangle KBC$ lie on the line KX.

Time: 4 hours.
Each problem is worth 8 points.

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Intermediate level

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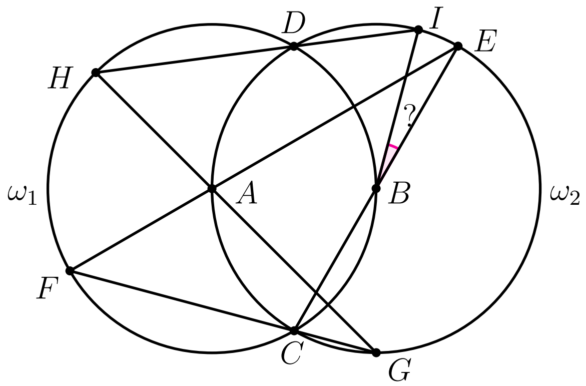
Problem 1. In the figure below points A, B are the centers of the circles ω_1, ω_2. Starting from the line BC points E, F, G, H, I are obtained respectively. Find the angle $\angle IBE$.



Problem 2. Points X, Y lie on the side CD of a convex pentagon $ABCDE$ with X between Y and C. Suppose that the triangles $\triangle XCB, \triangle ABX, \triangle AXY, \triangle AYE, \triangle YED$ are all similar (in this exact order). Prove that circumcircles of the triangles $\triangle ACD, \triangle AXY$ are tangent.

Problem 3. Let $\triangle ABC$ be an acute triangle with a point D on side BC. Let J be a point on side AC such that $\angle BAD = 2\angle ADJ$, and ω be the circumcircle of triangle $\triangle CDJ$. The line AD intersects ω again at a point P, and Q is the feet of the altitude from J to AB.
Prove that if $JP = JQ$, then the line perpendicular to DJ through A is tangent to ω.

Problem 4. Eric has assembled a convex polygon P from finitely many centrally symmetric (not necessarily congruent or convex) polygonal tiles. Prove that P is centrally symmetric.

Problem 5. Point P is the intersection of diagonals AC, BD of the trapezoid $ABCD$ with $AB \parallel CD$. Reflections of the lines AD and BC into the internal angle bisectors of $\angle PDC$ and $\angle PCD$ intersects the circumcircles of $\triangle APD$ and $\triangle BPC$ at D' and C'. Line $C'A$ intersects the circumcircle of $\triangle BPC$ again at Y and $D'C$ intersects the circumcricle of $\triangle APD$ again at X. Prove that P, X, Y are collinear.

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.

11$^{\text{th}}$ **Iranian Geometry Olympiad**

Advanced level

October 18, 2024

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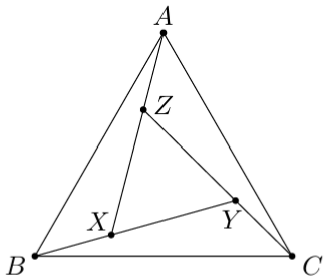
Problem 1. An equilateral triangle is split into 4 triangles with equal area; three congruent triangles $\triangle ABX, \triangle BCY, \triangle CAZ$, and a smaller equilateral triangle $\triangle XYZ$, as shown. Prove that the points X, Y, Z lie on the incircle of triangle $\triangle ABC$.



Problem 2. Point P lies on the side CD of the cyclic quadrilateral $ABCD$ such that $\angle CBP = 90°$. Let K be the intersection of AC, BP such that $AK = AP = AD$. H is the projection of B onto the line AC. Prove that $\angle APH = 90°$.

Problem 3. In the triangle $\triangle ABC$ let D be the foot of the altitude from A to the side BC and I, I_A, I_C be the incenter, A-excenter, and C-excenter, respectively. Denote by $P \neq B$ and $Q \neq D$ the other intersection points of the circle $\triangle BDI_C$ with the lines BI and DI_A, respectively. Prove that $AP = AQ$.

Problem 4. Point P is inside the acute triangle $\triangle ABC$ such that $\angle BPC = 90°$ and $\angle BAP = \angle PAC$. Let D be the projection of P onto the side BC. Let M and N be the incenters of the triangles $\triangle ABD$ and $\triangle ADC$ respectively. Prove that the quadrilateral $BMNC$ is cyclic.

Problem 5. Cyclic quadrilateral $ABCD$ with circumcircle ω is given. Let E be a fixed point on segment AC. M is an arbitrary point on ω, lines AM and BD meet at a point P. EP meets AB and AD at points R and Q, respectively, S is the intersection of BQ, DR and lines MS and AC meet at a point T. Prove that as M varies the circumcircle of triangle $\triangle CMT$ passes through a fixed point other than C.

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.