

XLI

IV

1. , 2000
 ,
 ?
 2000 : 4 = 500
 2000 - 500 = 1500
 1500 : 3 = 500
 1500 - 500 = 1000

2. ,
 13
 13
 13
 100 = 7 · 13 + 9,
 100 + 4 = 104.
 13
 13
 13 104, 114, 130, 143, 156, 169, 182

195.

:

- $13 \cdot (1 + 0 + 4) = 13 \cdot 5 = 65 \neq 104,$
 - $13 \cdot (1 + 1 + 7) = 13 \cdot 9 = 117,$
 - $13 \cdot (1 + 3 + 0) = 13 \cdot 4 = 52 \neq 130,$
 - $13 \cdot (1 + 4 + 3) = 13 \cdot 8 = 104 \neq 143,$
 - $13 \cdot (1 + 5 + 6) = 13 \cdot 12 = 156,$
 - $13 \cdot (1 + 6 + 9) = 13 \cdot 16 = 208 \neq 169,$
 - $13 \cdot (1 + 8 + 2) = 13 \cdot 11 = 143 \neq 183,$
 - $13 \cdot (1 + 9 + 5) = 13 \cdot 15 = 195,$
- 117, 156 185.

3. 34 cm.
 3 cm, 2 cm,

3 cm,

$$34 + 3 + 2 - 3 = 36 \text{ cm}.$$

$$36 : 3 = 12 \text{ cm}.$$

x .

a, b, c

$$a + b + c = 34,$$

$$a + 3 = x, b + 2 = x, c - 3 = x.$$

$$3x = a + 3 + b + 2 + c - 3,$$

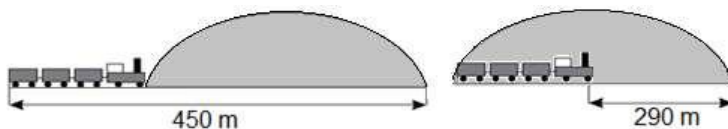
$$3x = (a + b + c) + 3 + 2 - 3,$$

$$3x = 34 + 3 + 2 - 3$$

$$3x = 36,$$

$$x = 12 \text{ cm}.$$

4.



x .

$$x + 290.$$

$$450 \text{ m},$$

$$x + x + 290 = 450,$$

$$2x + 290 = 450,$$

$$2x = 450 - 290,$$

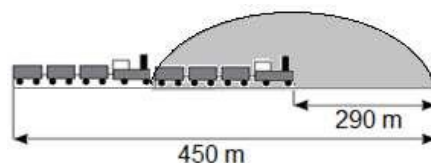
$$x = 80 \text{ m}.$$

80 m.

$$450 \text{ m} ($$

290 m

).



$$450 - 290 = 160 \text{ m},$$

$$160 : 2 = 80 \text{ m}.$$

V

1.

24

$\frac{3}{5}$

?

$\frac{3}{5}$

3

5

$$3 + 5 = 8$$

$$24 : 8 = 3$$

$$3 \cdot 3 = 9$$

$$3 \cdot 5 = 15$$

x

$\frac{3}{5}x$

$$x + \frac{3}{5}x$$

24,

$$x + \frac{3}{5}x = 24.$$

$$\frac{8}{5}x = 24,$$

$$8x = 24 \cdot 5,$$

$$x = (24 \cdot 5) : 8 = 15.$$

15

$$24 - 15 = 9$$

2.

$ABCD$

8 cm 5 cm

MN

20 cm .

MN .

$ABCD$

$$2 \cdot (8 + 5) = 26 \text{ cm} .$$

$AMND$

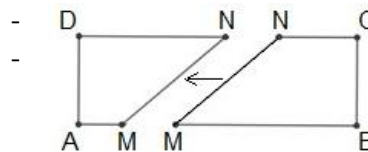
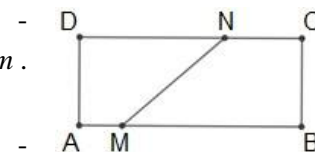
$MBCN$

$$2 \cdot 20 = 40 \text{ cm}$$

MN , () .

$$26 + 2\overline{MN} = 40, \dots \overline{MN} = (40 - 26) : 2 = 7 \text{ cm} .$$

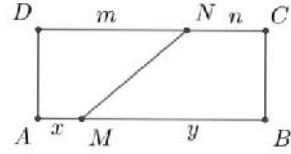
$$a = 8 \text{ cm} \quad b = 5 \text{ cm} .$$



$$a = x + y = m + n,$$

$$L_{AMND} = x + \overline{MN} + m + b,$$

$$L_{MBCN} = y + b + n + \overline{MN},$$



$$L_{AMND} + L_{MBCN} = x + \overline{MN} + m + b + y + b + n + \overline{MN}$$

$$= 2\overline{MN} + (x + y) + (m + n) + 2b$$

$$= 2\overline{MN} + 2(a + b),$$

$$20 + 20 = 2\overline{MN} + 2 \cdot (8 + 5), \dots \overline{MN} = (40 - 26) : 2 = 7 \text{ cm}.$$

3.

96 m.

$$a = 5b,$$

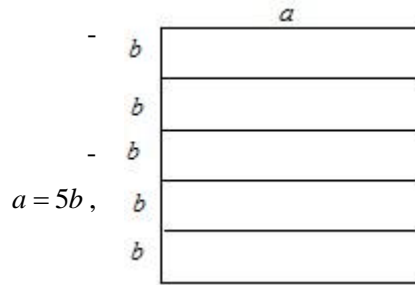
$$5 \cdot 2(a + b) = 10a + 10b.$$

$$2a = 2 \cdot 5b = 10b.$$

$$10a + 10b = 10a + 2a = 12a.$$

$$12a = 96, \dots a = 96 : 12 = 8 \text{ cm}.$$

$$4a = 4 \cdot 8 = 32 \text{ cm}.$$



$$a = 5b,$$

$$L = 4a,$$

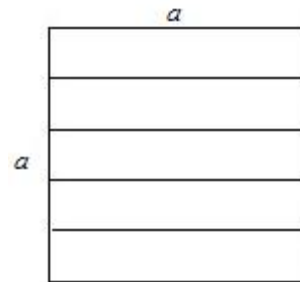
$$L = 4a,$$

$$L' = L + 4 \cdot 2a = L + 2 \cdot 4a = L + 2L = 3L.$$

$$L = 32 \text{ cm}.$$

$$3L = 96,$$

$$L = 32 \text{ cm}.$$



4.

92. ?

$$x, x+1, x+2, x+3, x+4, x+5, x+6, x+7$$

$$x + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6) + (x+7) = 92,$$

$$\dots 8x + 28 = 92, \quad x = (92 - 28) : 8 = 64 : 8 = 8.$$

8, 9, 10, 11, 12, 13, 14, 15.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36.$$

8

$$(92 - 36) : 8 = 56 : 8 = 7,$$

2, 3, 4, 5, 6, 7, 8, 9;

3, 4, 5, 6, 7, 8, 9, 10;

4, 5, 6, 7, 8, 9, 10, 11;

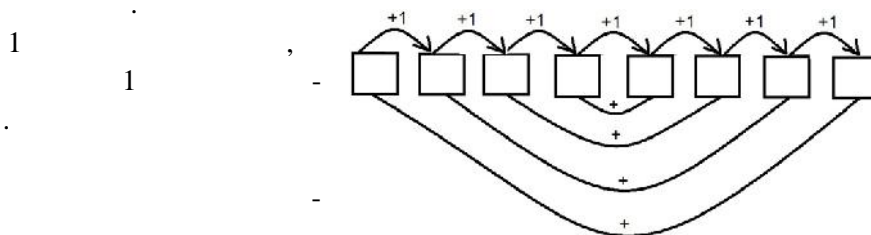
5, 6, 7, 8, 9, 10, 11, 12;

6, 7, 8, 9, 10, 11, 12, 13;

7, 8, 9, 10, 11, 12, 13, 14;

8, 9, 10, 11, 12, 13, 14, 15

8, 9, 10, 11, 12, 13, 14, 15.



4

$$92 : 4 = 23.$$

, $11+12 = 23$, $11 \quad 12$,
 8, 9, 10, 11, 12, 13, 14 15.

VI

1. 12 . -
 3 .
 12
 11 , -
 $X \quad Y$ $Y \quad X$, -
 $12 \cdot 11 = 132$. ,
 $\frac{12 \cdot 11}{2} = 66$. , 3
 $3 \cdot 66 = 198$.

2. 63000 ? -
 63000 .
 $63000 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7 = 2^3 \cdot 3^2 \cdot 5^3 \cdot 7$. (1)
 $63000 \cdot 70 = 2^4 \cdot 3^2 \cdot 5^4 \cdot 7^2 = (2^2 \cdot 3 \cdot 5^2 \cdot 7)^2 = 2100^2$.

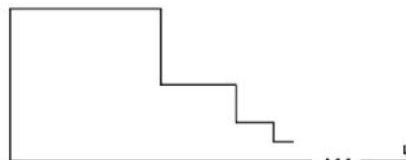
3. 2023 cm
 1309 cm . -
 a -
 $2023 \quad 1309$, a

$a = \text{NZD}(2023, 1309) = 119 \text{ cm} .$

$, 2023 : 119 = 17 \quad 1309 : 119 = 11 ,$
 $17 \cdot 11 = 187 \quad a = 119 \text{ cm} .$
 $L = 187 \cdot 4a = 187 \cdot 4 \cdot 119 = 89012 \text{ cm}$

4.

1024 cm .



(
)
 .
)
 .
 .)

: 1024 cm, 512 cm, 256 cm, 128 cm, 64 cm, 32 cm, 16 cm, 8 cm, 4 cm, 2 cm 1 cm ,

$$L = 2 \cdot 1024 + 2(1024 + 512 + 256 + 128 + 68 + 32 + 16 + 8 + 4 + 3 + 2 + 1)$$

$$= 2048 + 2 \cdot 2047 = 6142 \text{ cm}.$$

) $\frac{11}{32} \text{ cm} .$,

$$P = 32 \cdot 32 = 1024 \text{ cm}^2 .$$

VII

1.

$$1 - \frac{100}{101} + \frac{99}{101} - \frac{98}{101} + \frac{97}{101} - \dots - \frac{2}{101} + \frac{1}{101} .$$

∴ :

$$1 - \frac{100}{101} + \frac{99}{101} - \frac{98}{101} + \frac{97}{101} - \dots - \frac{2}{101} + \frac{1}{101} = \frac{101}{101} - \frac{100}{101} + \frac{99}{101} - \frac{98}{101} + \frac{97}{101} - \dots - \frac{2}{101} + \frac{1}{101}$$

$$= \frac{101-100}{101} + \frac{99-98}{101} + \frac{97-96}{101} + \dots + \frac{3-2}{101} + \frac{1}{101}$$

$$= \underbrace{\frac{1}{101} + \frac{1}{101} + \dots + \frac{1}{101}}_{50} + \frac{1}{101} = \frac{51}{101}.$$

2. 2023-

$$\frac{35}{37}.$$

$$\frac{35}{37} = 0,(945)$$

$$2023 = 674 \cdot 3 + 1$$

2023-

9.

3.

360

384

60

?

x

$$, x|360 \quad x|384,$$

$$x|NCD(360,384) = 24.$$

$$x = 24,$$

$$260 : 24 = 15$$

$$384 : 24 = 16$$

$$15 + 16 = 31 < 60$$

$$24 < 12,$$

$$x \leq 12,$$

$$360 : x \geq 360 : 12 = 30$$

$$384 : x \geq 384 : 12 = 32$$

$$32 + 30 = 62 > 60,$$

15

16

24

30

4.

ABC ,

$$\overline{AC} = \overline{BC}.$$

AC

M

N

$$\sphericalangle MBA = \sphericalangle CBN$$

$$\overline{MN} = \overline{BM},$$

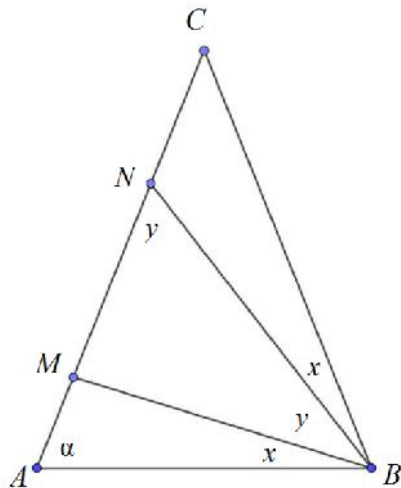
M

A

N .

$$\sphericalangle NBA.$$

$\angle BAC = \angle CBA = r,$
 $\angle MBA = \angle CBN = x$ $\angle NBM = y.$
 $r = 2x + y.$ BNM
 $\angle NBM = \angle MNB = y.$
 ABN $2x + y, x + y, y,$
 $2x + y + x + y + y = 180^\circ,$
 $x + y = 60^\circ.$
 $\angle NBA = x + y = 60^\circ.$



VIII

1.

8, 10 13

39

?

x
 $x - 8, x - 10 \quad x - 13$
 $x + 1$ $x - 7, x - 9$

$x - 12 \quad x + 1 + 39 = x - 7 + x - 9 + x - 12,$

$x = 34.$

x
 $x - 8, x - 10 \quad x - 13$

3,

1,

$39 - 3 + 1 = 37$

$x - 8 + x - 10 + x - 13 = x + 37,$

$x = 34.$

2.

2400

$\frac{3}{5}$

2:3,

5:4:6.

a, b, c, d, e
 $x = a + b \quad y = b + c + d. \quad x = \frac{3}{5}y$
 $x + y = 2400, \quad \frac{3}{5}y + y = 2400, \quad \dots \quad y = 1500$
 $x = 2400 - 1500 = 900. \quad a : b = 2 : 3 \quad a + b = 900$
 $a = \frac{900}{2+3} \cdot 2 = 360 \quad b = \frac{900}{2+3} \cdot 3 = 540. \quad c : d : e = 5 : 4 : 6 \quad c + d + e = 1500,$
 $c = \frac{1500}{5+4+6} \cdot 5 = 500, \quad d = \frac{1500}{5+4+6} \cdot 4 = 400 \quad e = \frac{1500}{5+4+6} \cdot 6 = 600.$
 360
 $540 \quad , \quad 500, \quad 400 \quad 600$
 a, b, c, d, e

$$\begin{cases} a + b + c + d + e = 2400, \\ a + b = \frac{3}{5}(c + d + e), \\ a : b = 2 : 3, \\ c : d : e = 5 : 4 : 6. \end{cases}$$

$$a = 2k, b = 3k, c = 5m, d = 4m, e = 6m$$

$$5k + 15m = 2400$$

$$5k = \frac{3}{5} \cdot 15m, \quad \dots \quad 5k + 15m = 2400 \quad 5k = 9m.$$

$$24m = 2400, \quad \dots \quad m = 100$$

$$5k = 900, \quad \dots$$

$$k = 180.$$

$$a = 360, b = 540, c = 500, d = 400$$

$$e = 600.$$

$$360 \quad , \quad 540, \quad 500 \quad 400 \quad 600$$

3.

ABCD

$$\overline{AB} = 2\overline{BC}.$$

CD

M,

$\angle DMB$

A.

$\angle AMB$.

$$\overline{AB} = a, \overline{BC} = b.$$

$$\overline{AB} = 2b.$$

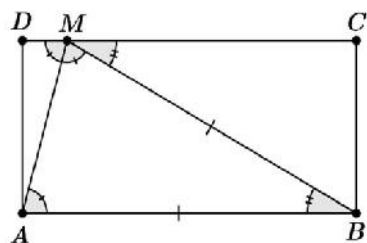
AM

$\angle DMB,$

$$\angle DMA = \angle AMB.$$

$$\angle DMA = \angle BAM,$$

$$\angle BAM =$$



$$\begin{aligned} \angle AMB, & \quad \overline{BM} = \overline{AB} = 2b, \\ & \quad \overline{BC} = b, \\ & \quad \overline{BM} = 2b, \\ \angle BMC = 30^\circ, & \quad \angle CBM = 60^\circ, \\ & \quad \angle DMB = 180^\circ - \angle BMC = 180^\circ - 30^\circ, \\ \angle AMB = \frac{1}{2} \angle DMB = \frac{150^\circ}{2} = 75^\circ. \end{aligned}$$

4.

$$\begin{aligned} \sqrt{\frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}}}, & \quad \sqrt{\frac{x^2 + 20}{x^2 - 20}} - \sqrt{\frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}}} \\ & \quad \sqrt{\frac{x^2 + 20}{x^2 - 20}} \sqrt{\frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}}} \\ & \quad \frac{x^2 + 20}{x^2 - 20} \frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}} = \frac{5y^3 + 24}{5y^3 - 24}, \\ \frac{x^2 + 20}{x^2 - 20} = \frac{x^2 - 20 + 40}{x^2 - 20} = 1 + \frac{40}{x^2 - 20}, & \quad x^2 - 20 > 0, \\ x^2 = 20 + k, \quad k | 40, \quad k \in \{1, 2, 4, 5, 8, 10, 20, 40\} & \\ k = 5 & \\ x = 5 \sqrt{\frac{x^2 + 20}{x^2 - 20}} = 3. & \\ \frac{5y^3 + 24}{5y^3 - 24} = 1 + \frac{48}{5y^3 - 24}, & \quad 5y^3 - 24 > 0, \\ y^3 = \frac{24 + m}{5}, \quad m | 48. \quad m \in \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\} & \\ m = 16 & \\ y = 2 \sqrt{\frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}}} = \sqrt{\frac{5y^3 + 24}{5y^3 - 24}} = 2. & \\ \sqrt{\frac{x^2 + 20}{x^2 - 20}} - \sqrt{\frac{y^3 + \frac{24}{5}}{y^3 - \frac{24}{5}}} = 3 - 2 = 1^2, & \end{aligned}$$

$$x=5 \quad y=2.$$

IX

1.

a, b, c, d, e

$ab, ac, ad, ae, bc, bd, be, cd, ce, de$.

$\sqrt{ab}, \sqrt{ac}, \sqrt{ad}, \sqrt{ae}, \sqrt{bc}, \sqrt{bd}, \sqrt{be}, \sqrt{cd}, \sqrt{ce}, \sqrt{de}$,

$$\begin{aligned} A &= \sqrt{\sqrt{ab}\sqrt{ac}\sqrt{ad}\sqrt{ae}\sqrt{bc}\sqrt{bd}\sqrt{be}\sqrt{cd}\sqrt{ce}\sqrt{de}} \\ &= \sqrt{\sqrt{(ab)(ac)(ad)(ae)(bc)(bd)(be)(cd)(ce)(de)}} \\ &= \sqrt[4]{a^4 b^4 c^4 d^4 e^4} = \sqrt[4]{(abcde)^4} = abcde. \end{aligned}$$

$$a = 2, b = 3, c = 5, d = 7, e = 11,$$

$$A = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2310.$$

2.

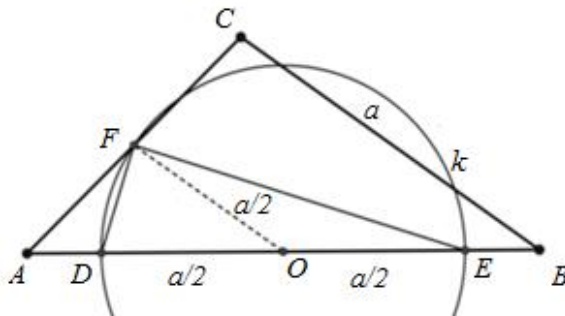
$AB \quad \overline{ABC}, \overline{AB} > \overline{BC}$

$D \quad E \quad \overline{DE} = \overline{BC} \quad \overline{AD} = \overline{BE} \quad (D \quad A \quad E).$

$F \quad AC, \quad \angle DFE = 90^\circ.$

$O \quad AB, \dots \overline{AO} = \overline{BO}$

$\overline{DE} = \overline{BC} = a$ ().



$$\begin{aligned} \overline{DO} &= \overline{AO} - \overline{AD} = \overline{BO} - \overline{BE} = \overline{EO}, & O \\ &DE, & \overline{DO} = \overline{EO} = \frac{a}{2}, & O \\ &AB \quad F & AC, & FO \\ &ABC, & \overline{OF} = \frac{\overline{BC}}{2} = \frac{a}{2}, & \\ \overline{DO} = \overline{EO} = \overline{OF} &= \frac{a}{2} & F & k \\ &DE. & \angle DFE & \\ &DE, & \angle DFE = 90^\circ & \end{aligned}$$

3. 1500 . 1300 1400 , 1400
 6- , 110 .
 7- .
 . $36^2 = 1296, 37^2 = 1369, 38^2 = 1444, 39^2 = 1521,$
 1500 1444. , 1300 1400 1369, 1400
 1369 , 1444

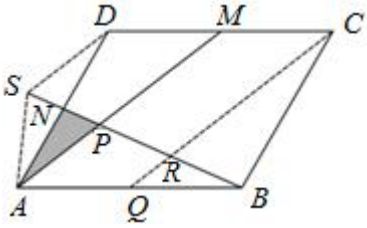
7- ,
 $m^2 < 111, n^2 < 111, m, n \in \mathbb{N}.$
 m^2 $1369 + m^2$, n^2
 $1444 + n^2$.

$1369 + m^2 = 1444 + n^2, \dots m^2 - n^2 = 75.$
 , 11 1, 4, 9, 16, 25, 36, 49, 64, 81 100 -
 75 $m^2 = 100$ $n^2 = 25.$
 $1369 + 10^2 = 1469$ 100, 25 .
 $m^2 - n^2 = 75$

$(m - n)(m + n) = 1 \cdot 75 = 3 \cdot 25 = 5 \cdot 15,$
 , $0 < m - n < m + n,$
 , $m = 38, n = 37, m = 14, n = 11 \quad m = 10, n = 5,$
 $m = 10, n = 5$

111.

4. $ABCD$. M
 CD , N AD . AM BN
 P .
 $ABCD$ ANP ?
 S
 BN , N AD PS
 PS .
 $APDS$
 Q
 AB . $\overline{AQ} = \overline{MC}$ $AQ \parallel MC$
 $AQCM$
 $CQ \parallel MA \parallel DS$. , $\overline{BQ} = \overline{QA}$ $\overline{BR} = \overline{RP}$, $\overline{CM} = \overline{MD}$
 $\overline{RP} = \overline{PS}$. , $\overline{BR} = \overline{RP} = \overline{PS}$. , $\overline{PS} = 2\overline{PN}$,
 $\overline{BN} = \overline{BR} + \overline{RP} + \overline{PN} = 2\overline{PN} + 2\overline{PN} + \overline{PN} = 5\overline{PN}$.
 $h_B = 5h_p$, h_B , h_p
 ANP . ,
 $P_{ABCD} = \overline{AD} \cdot h_B = 2\overline{AN} \cdot 5h_p = 20 \cdot \frac{\overline{AN} \cdot h_p}{2} = 20P_{APN}$,
 \dots



$$P_{APN} = \frac{1}{20} P_{ABCD}.$$