

2-nd Mediterranean Mathematical Competition 1999

1. Do there exist a circle and an infinite set of points on it such that the distance between any two of the points is rational?
2. A plane figure of area $A > n$ is given, where n is a positive integer. Prove that this figure can be placed onto a Cartesian plane so that it covers at least $n+1$ points with integer coordinates.
3. Let a, b, c be nonzero numbers and x, y, z be arbitrary positive numbers with $x+y+z=3$. Prove that inequality

$$\frac{3}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} \geq \frac{x}{1+a^2} + \frac{y}{1+b^2} + \frac{z}{1+c^2}.$$

4. In a triangle ABC with $BC = a$, $CA = b$, $AB = c$ we have $\angle B = 4\angle A$. Show that

$$ab^2c^3 = (b^2 - a^2 - ac)((a^2 - b^2)^2 - a^2c^2).$$